to constants of integration.[4] By making obvious scale transformations, B_0, C_0, and D_0 may be eliminated from the line element, b_1 surviving to parametrize the solution.

Hence, the line element (2), representing a solution which is obtained either by specializing the author's line element (1) or by specializing Brill's line element (3), is common to both spatially homogeneous solutions to the Einstein-Maxwell equations, even though the two solutions are generally inequivalent. On the one hand, the author's line element (1) represents a more general solution that is not necessarily symmetrical with respect to rotations about a preferred axis (the x_1 axis). On the other hand, Brill's line element (3) represents a more general solution that is not necessarily Euclidean in the hypersurfaces of constant time, but symmetrical with respect to rotations about a preferred axis (the z axis). The fact that the two spatially homogeneous solutions were obtained originally from very different formulations of the Einstein-Maxwell equations, using very different integration procedures, illustrates the value of our having and studying alternative formulations of field equations in general relativity, such as the Rainich and Cartan formulations of the Einstein-Maxwell equations.

[4] The difference in the constants of integration is mainly due to the limiting procedure $R_0 \to \infty$, but also due in part to the fact that B(21) is not the *general solution* of the nonlinear total differential equation

$$d^2B/dt''^2 = B_0^2/B^3$$

supplemented with the initial condition $B = B_0$ at $t'' = t_0''$, the general solution being $B^2 = B_0^2 + 2kB_0(t''-t_0'') + (1+k^2)(t''-t_0'')^2$ with k a new and free constant of integration; Eq. B(21), the *particular solution* with $k=0$, is not as general as it ought to be.

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Origin of the Light Elements

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It is shown that the observed isotopic abundances of Li, Be, and B can be explained by their spallation in small, prototerrestrial bodies. Spheres of arbitrary composition and radius are irradiated by protons; approximate expressions are found for the solar-flare proton spectrum, the spallation cross sections, and neutron production. A new approximation is made for the effect of the fast neutrons. It is then found that the present day proton flux is too soft to give the desired results reasonably, and that a mean proton energy of 300 MeV is necessary to get the observed isotopic ratios. The results are not sensitive to the composition, and we can obtain the measured Li, Be, and B abundances by taking dry silicate spheres of about 140 m for the protoasteroidal bodies. In order to obtain the observed D/H ratio from the irradiation, however, it is necessary to add 10% H_2O. The measured crustal abundances of Li, Be, B lead to different values for D/H and for the depletion of Gd^{157} for the earth and for asteroids, contrary to observation. These discrepancies disappear if we assume that Li, Be, and B have been concentrated tenfold in the earth's crust. The different isotopic ratios found for terrestrial and meteoritic material are consistent with this model, and enable us to calculate the Li^7/Li^6 ratios to be expected on the other planets.

INTRODUCTION

THE origin of the light elements H^2, Li, Be, and B has long been a puzzle to cosmologists.[1,2] It has been suggested that they were produced by spallation reactions[2] on heavier elements. A reasonable such model was proposed by Fowler, Greenstein, and Hoyle[3]: Energetic solar protons spallated the light isotopes in solid bodies, and neutrons, simultaneously produced, bathed the resulting nuclei, tending to create the observed abundances.

The astrophysical setting posited by FGH is also assumed here: A rather cool protosun extremely active in emitting energetic protons, the solar system outgassed and inhabited by relatively small solid objects (protoplanets or planetesimals) orbiting the protosun and being irradiated by it.

There are two principal differences between the calculation presented in this paper and the one undertaken by FGH: First, they work backward from the presently observed isotopic abundances to an inferred "intermediate stage" in the evolution of the solar system. Here, we start from several plausible intermediate stages and follow the results of the solar proton bombardment forward. Secondly, FGH uses the mean value

[1] R. A. Alpher and R. C. Herman, Rev. Mod. Phys. 22, 153 (1950).
[2] E. M. Burbidge, G. R. Burbidge, W. A. Fowler, and F. Hoyle, Rev. Mod. Phys. 29, 547 (1957); W. A. Fowler, G. R. Burbidge, and E. M. Burbidge, Astrophys. J. Suppl. 2, 167 (1955). S. Bashkin and D. C. Peaslee, Astrophys. J. 134, 981 (1961).
[3] W. A. Fowler, J. L. Greenstein, and F. Hoyle, Geophys. J. 6, 148 (1962)—hereafter referred to as FGH; Am. J. Phys. 29, 393 (1961).

of the theoretical proton spectrum from solar flares derived by E. N. Parker[4]; this is equivalent to using a delta-function spectrum. Since this mean is high (several hundred MeV), the spallation cross sections for formation of the various nuclei are all in the asymptotic region where they are independent of energy. Instead, we use an approximate observed spectrum and fold this into the excitation functions for spallation, to obtain the effective production. We ask for the nuclear consequences of such irradiation on solid spheres of arbitrary composition.

The object of the calculation is to see whether we can approach a consistent picture for the formation of the light elements, and thereby for the evolution of the earth and perhaps the solar system. Hopefully, the various abundance measurements can fix the composition and the distribution of particle sizes for a hypothetical set of preplanetary bodies which agglomerated to form the earth.

I. IRRADIATION OF THE SPHERES

The limitation to spheres is not very important. This can be seen by noting that objects of differing shapes can be approximated by spheres—thus, a long right circular cylinder can be replaced by a chain of contiguous spheres of equal volume, i.e., if the radius of the cylinder is a, the radius of the equivalent spheres is $R=(\frac{3}{2})^{1/2}a$. Of course, the irradiation of such objects will be different to some extent from that of the *free* spheres, but since in any case we will eventually arrive at a size distribution, such effects will be automatically "corrected" for. The probability of large deviations from sphericity is small in any case, since every dynamical process will on the average favor a minimum-surface body.

The high-energy protons impinge on the sphere, spallating its nuclei and giving rise to light fragments, including neutrons.

Suppose we have a flux φ_p of protons with some energy spectrum, incident on a sphere of radius R, homogeneous but of arbitrary composition. We wish to know the total amount of the various spallated elements, e.g., B^{10}, Li^6, at the end of t years.

We assume a static model, i.e., the sphere does not grow larger or smaller during the period of synthesis, and the astrophysical environment does not change significantly.

The density of a nonvolatile component a at the point $\mathbf{r}$ in the sphere at time t, $N_a(\mathbf{r},t)$ is given by

$$\partial N_a(\mathbf{r},t)/\partial t=S_a(\mathbf{r})-L_a(\mathbf{r},t). \tag{1}$$

The relation

$$S_a=\varphi_p(\mathbf{r})\sigma_a{}^s n \tag{2}$$

gives the direct production of a by spallation, where φ_p is the proton flux at $\mathbf{r}$, $\sigma_a{}^s$ is the spallation cross section of a, and n is the target number density. L_a is the loss rate; for a nondiffusing element, it is just the rate of destruction by neutrons. There is also some destruction by the proton flux, but we will neglect it here. Hence,

$$L_a(\mathbf{r},t)=\varphi_n(\mathbf{r},t)\sigma_{at}N_a(\mathbf{r},t), \tag{3}$$

where φ_n is the effective neutron flux and σ_{at} the thermal-neutron absorption cross section of nucleus a. For a homogeneous body, σ_{at} is independent of position, and we will see that the effective neutron flux φ_n is independent of time.

In general, call

$$\beta_a(\mathbf{r},t)\equiv\sigma_{at}\varphi_n(\mathbf{r},t).$$

Dropping the subscript for the moment, we find the solution of Eq. (1):

$$N(\mathbf{r},t)=S(\mathbf{r})\exp\left(-\int\beta dt\right)\int_0^t\exp\left(\int\beta dt\right)dt. \tag{4}$$

The neutron distribution achieves equilibrium in a few relaxation times, which are of the order

$$t_R\sim R^2/10D_0,$$

where D_0 is the diffusion constant.

At a temperature of 170°K, $D_0\sim3\times10^4$ cm^2/sec, for a typical[5] icy silicate, so that

$$t_R\sim3R^2\times10^{-6}\text{ sec}.$$

Thus, even for a large object, the relaxation time is very short compared with the assumed duration of synthesis, 10^7 yr. Moreover, we assume (as a first approximation) that the spallated isotopes with large thermal neutron absorption cross sections will be produced in sufficiently small amounts that the neutron flux distribution will be unaffected by them.[5] We may then take β as independent of time, and Eq. (4) can be immediately integrated:

$$N(\mathbf{r},t)\cong\frac{S(\mathbf{r})}{\beta(\mathbf{r})}[1-e^{-\beta(\mathbf{r})t}]. \tag{5}$$

We make the further simplifying assumption that the spheres rotate at random, so that all the distributions in the average sphere are isotropic.

The unaveraged distribution (5) would be particularly interesting when making isotopic examinations of unmetamorphosed meteorites—possibly primeval bits of matter.[6] It could also be useful in giving an idea of the results of cosmic-ray spallation effects, and of the amounts of shielding ablated away from meteorites upon atmospheric entry.

On the other hand, if there has been thorough mixing

[4] E. N. Parker, Phys. Rev. **107**, 830 (1957).

[5] H. E. Mitler, J. Geophys. Res. **68**, 4587 (1963).

[6] H. C. Urey and H. Craig, Geochim. Cosmochim. Acta **4**, 36 (1953).

of the spallated nuclei, with no subsequent fractionation, the *average* composition is more interesting.

The total amount produced at time t is

$$\int_V N(r,t)dV \cong 4\pi \int_0^R \frac{S(r)}{\beta(r)}[1-e^{-\beta(r)t}]r^2 dr.$$

The production is confined to a thin shell at the surface; just there, however, the neutron flux falls off to zero and will therefore be smaller than that calculated by FGH. Thus, the neutrons will play a smaller role, reducing less, for example, the Li^6 abundance. The neutrons, on the other hand, will diffuse throughout the sphere and subject *all* nuclei to some neutron flux, possibly depleting isotopes of large thermal neutron absorption cross sections. We shall return to this in Sec. VI.

The production of nucleus a at the depth x is just

$$S_a(x) = n\sigma_a \varphi_p(x)\,, \tag{6}$$

where $\varphi_p(x)$ is the proton flux at the depth x, σ_a is the production cross section, and n is the number density of targets. If E_x is the energy necessary to penetrate to the depth x, the proton flux at depth x is

$$\varphi_p(x) = e^{-x/\lambda} \int_{E_x}^{\infty} \varphi_p(0,E)dE\,, \tag{7}$$

where λ is the proton mean free path, and where for simplicity's sake we have assumed λ to be independent of E. From $\varphi_p(x)$ we can then calculate the effective isotropic proton flux in a sphere.

The range of a proton in matter at the interesting energies is[7]

$$R(E) \cong \alpha' E^{1.6}\,,$$

and we approximate this by

$$R(E) \cong \alpha E^{1.5}$$

for the sake of convenience. Thus, x and E_x are related by

$$x \cong \alpha E_x^{1.5}. \tag{8}$$

The *productive* flux, on the other hand, must be

$$\tilde{\varphi}_p(x) = e^{-x/\lambda} \int_{E_x+\delta}^{\infty} \varphi_p(0,E)dE\,, \tag{9}$$

where $\delta = \alpha E_t^{1.5}$, with E_t the threshold for production.

Generally, of course, the proton spectrum will not be monoenergetic. Therefore, if we write the production due to incident protons of energy E as $S_a(r,E)$ and the neutron flux due to these protons as $\varphi_n(r,E)$, Eq. (1) will still hold, with (2) and (3) replaced by their integrals over the energy.

[7] R. B. Leighton, *Principles of Modern Physics* (McGraw-Hill Book Company, Inc., New York, 1959), p. 734.

II. PARTICLE AND ENERGY FLUXES FROM SOLAR-FLARE PLASMAS

Different solar events give rise to significantly different particle spectra; if we try to fit them to a power law of the form

$$\varphi_p(E) = k/E^\gamma\,, \tag{10}$$

we find[8]

$$3 < \gamma < 6.$$

The spectra change with time as well, but we want the time-averaged distribution. For the 30 major events in the 6 active years 1956–1961, McDonald gives[9]

$$N(E > 30 \text{ MeV}) \cong 2.17 \times 10^{10} \text{ protons/cm}^2\,,$$
$$N(E > 100 \text{ MeV}) \cong 1.79 \times 10^{9} \text{ protons/cm}^2.$$

Fitting these to (10), we find $\gamma = 3.07$ and $k = 1.9 \times 10^5$ MeV^2/cm^2 sec, i.e.,

$$k = 0.30 \text{ MeV erg/cm}^2 \text{ sec}.$$

Assuming we have a low-energy cutoff E_m, we find that the energy flux corresponding to this spectrum, with $\gamma = 3$, is

$$\varphi_E = \int_{E_m}^{\infty} E\varphi_p(E)dE = \frac{k}{(\gamma-2)E_m{}^{\gamma-2}} = \frac{0.30}{E_m} \text{ erg/cm}^2 \text{ sec}, \tag{11}$$

where E_m is in MeV. The particle flux is

$$\varphi_p = \int_{E_m}^{\infty} \varphi_p(E)dE = \frac{k}{(\gamma-1)E_m{}^{\gamma-1}} \cong \frac{0.15}{E_m{}^2} \text{ protons/cm}^2 \text{ sec}, \tag{12}$$

and from (11) and (12), the mean energy per proton is

$$\bar{E} = [(\gamma-1)/(\gamma-2)]E_m \cong 2E_m.$$

The energy flux from the solar wind at 1 au is of the order[10] 0.2 erg/cm^2 sec. Protons from highly energetic solar flares, on the other hand, contribute some 0.005 erg/cm^2 sec in intermittent bursts.[9]

In order to have such an energy flux from Eq. (11), we need $E_m \cong 60$ MeV. The mean value, $\bar{E} = 2E_m \cong 120$ MeV, is not unreasonable, but since we observe significant fluxes even below 1 MeV in these energetic showers, this lower bound is unphysical, and hence a power law is not very satisfactory.

Parker[4] has given a theoretical expression for the integral spectrum: Calling

$$X(E) \equiv 1 + E + \tfrac{1}{2}E^2\,,$$

[8] C. E. Fichtel, D. A. Kniffen, and K. W. Ogilvie, J. Geophys. Res. 67, 3669 (1962). A. J. Masley, T. C. May, and J. R. Winckler, *ibid.* 67, 3243 (1962).

[9] F. B. McDonald, Goddard Space Flight Center, National Aeronautics and Space Administration Technical Report TR-R-169, 1963 (unpublished).

[10] M. Neugebauer and C. W. Snyder, Science 138, 1095 (1962).

his integral spectrum is

$$N(\epsilon>E)=N_0X^{-2}, \tag{13}$$

where the kinetic energy E of the proton is in units of $m_nc^2=939$ MeV. Thus,

$$\varphi_p(E)=2N_0(E+1)X^{-3}$$

and

$$\bar{E}=2\int_0^\infty E\varphi(E)dE=\frac{\pi}{2}-1\cong 540 \text{ MeV}.$$

Not only does this look very high, but expression (13) fails to fit the two data points from McDonald. More generally, Freier and Webber[10a] have shown that a spectrum exponential in rigidity,

$$J=J_0e^{-P/P_0},$$

fits all the data very well, down to energies as low as 20 MeV. Making the reasonable assumption that it holds down to still lower energies, the mean rigidity is $\bar{P}=P_0$. The flattest (i.e., most energetic) spectrum they measured was for the 15 November 1960 flare, which was fitted with $P_0=375$ MV, corresponding to a mean energy 72 MeV.

For our purposes, however, the *exact* shape of the spectrum is not very important. Because of Eqs. (7) and (8), it would be analytically convenient to fit the data to a function of the form

$$\varphi_p(E)\cong pE^{3\mu/2}\exp(-qE^{1.5}).$$

If we make the simple choice $\mu=\frac{1}{3}$, then

$$N(E>\epsilon)=\int_\epsilon^\infty \varphi(E)dE=(2p/3q)e^{-qE^{3/2}}. \tag{14}$$

Fitting this to McDonald's data we find

$$q\cong 3.0\times 10^{-3}\text{ MeV}^{-3/2}$$

and

$$p\cong 0.84\text{ protons/MeV}^{3/2}\text{ cm}^2\text{ sec}.$$

(Since McDonald's compilation was for the active half of the solar cycle, we would do well to take $\frac{1}{2}$ to $\frac{2}{3}$ of this value for p.) This value of q gives a maximum for φ_p at $E_{\max}=(1/3q)^{2/3}\cong 23$ MeV. The total energy flux would then be

$$\varphi_E=\frac{2p}{3q^{5/3}}\Gamma(5/3)\cong\frac{p}{q^{5/3}}, \tag{15}$$

and with the above values for p and q,

$$\varphi_E\cong 0.022\text{ erg/cm}^2\text{ sec},$$

which is not inconsistent with the estimates made above. Also, the total particle flux, from (14), is

$$\varphi_p(0)=N(E>0)=\frac{2p}{3q}\cong 0.19\text{ protons/cm}^2\text{ sec}, \tag{16}$$

and the mean energy per proton is

$$\bar{E}=\varphi_E/N\cong 1.5q^{-2/3}\cong 72\text{ MeV/proton}. \tag{15$'$}$$

With the proton flux

$$\varphi_p(0,E)=pE^{1/2}\exp(-qE^{1.5})$$

at the surface, and Eq. (7), we can now calculate the flux at any depth. Defining

$$w\equiv E^{1.5},$$

then from Eq. (9), we find

$$\tilde{\varphi}_p(x)=\varphi_p(0)e^{-qw_t}e^{-kx}, \tag{17}$$

where

$$k=\frac{1}{\lambda}+\frac{q}{\alpha} \tag{18}$$

and the incident flux is $\varphi_p(0)=2p/3q$. The production is then given by Eq. (6). The equivalent isotropic production in a sphere of radius R is

$$S_a(r)=\bar{S}_aF(r)$$

at the radius r, where

$$\begin{aligned}F(r)=(1/4rk)\{&(kR+kr+1)e^{-k(R-r)}\\&-(kR-kr+1)e^{-k(R+r)}+k^2(R^2-r^2)\\&\times[\mathrm{Ei}[-k(R-r)]-\mathrm{Ei}[-k(R+r)]]\},\end{aligned} \tag{19}$$

and

$$\bar{S}_a=(2p/3q)\bar{n}\bar{\sigma}_s{}^a \tag{20}$$

is just the production at the front surface. Equation (19) may be compared with the simpler phenomenological expression used by Eberhardt *et al.*[11]

III. SPALLATION AND NEUTRON PRODUCTION

A considerable amount of experimental[12] and theoretical[13] work has been done on medium and high-energy spallation of nuclei; yet it is impossible to find the excitation functions for most light fragments off most targets, in the literature. However, there are enough data available so that we can make estimates from systematics.

[10a] P. S. Freier and W. R. Webber, J. Geophys. Res. **68**, 1605 (1963).

[11] P. Eberhardt, J. Geiss, and H. Lutz, in *Earth Science and Meteoritics*, edited by J. Geiss and E. D. Goldberg (North-Holland Publishing Company, Amsterdam, 1963), p. 143.

[12] J. M. Miller and J. Hudis, Ann. Rev. Nucl. Sci. **9**, 159 (1959). G. Rudstam, Doctoral thesis NP-619 University of Uppsala, 1956 (unpublished).

[13] N. Metropolis, R. Bivins, M. Storm, A. Turkevich, J. M. Miller, and G. Friedlander, Phys. Rev. **110**, 185 (1958); **110**, 204 (1958); I. Dostrovsky, P. Rabinowitz, and R. Bivins, *ibid.* **111**, 1659 (1958); H. W. Bertini, *ibid.* **131**, 1801 (1963).

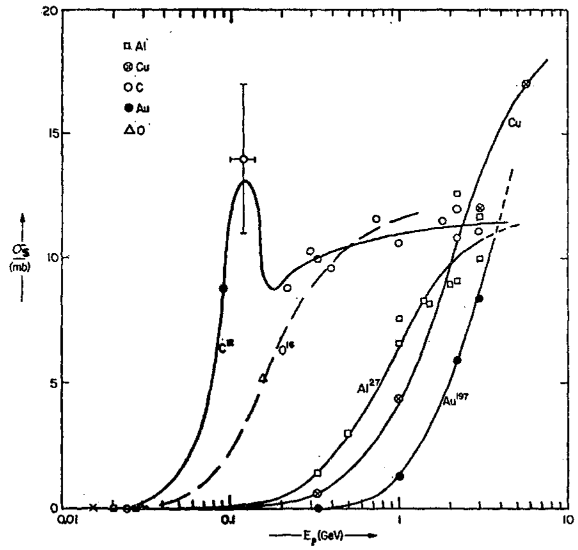

FIG. 1. Excitation functions for the spallation of Be^7 off C, O, Al, Cu, and Au. Most of the measurements are $\pm 25\%$. The most thoroughly measured target element has been C. The dashed curve is the function for O^{16}; it is assumed to be similar to the other curves and goes through the one measured point.

Be^7 is radioactive, and hence is one of the best-studied spallation nuclei.[12,14,15] We find for it that the production cross section is an increasing function of energy, but the trend with A is more complicated. At low energies, σ decreases with A because of the increasing Coulomb barrier, while at high energies it increases because of the larger number of available nucleons. An adequate expression for the formation cross section for Be^7 at proton-bombarding energy E (MeV) off target of mass A is

$$\log_{10}\sigma = 9.62 - 2.6 \log_{10}E - (8 - 2.4 \log_{10}E) \log_{10}A\,, \qquad (21)$$

with σ in mb. This is not very convenient for analytic handling, however. The excitation functions off different targets are all quite similar and can be fitted by a curve of the form

$$\sigma = \sigma_\infty f(x - x_0)\,,$$

where

$$x = \log_{10}E \quad \text{and} \quad x_0 = \log_{10}E_t\,,$$

E_t being an (effective) threshold energy for the reaction. Very roughly, for Be^7 production off various targets, we have

$$E_t \sim 1.5A \text{ MeV} \qquad (22)$$

(see Fig. 1). The cross section reaches an asymptotic value σ_∞ at high energies, which is only slightly dependent on A; it is about 12 mb for light targets and increases slowly. Thus, for Be^7, a fair approximation is

$$\sigma_\infty \cong 12(A/12)^{1/4} \text{ mb}.$$

We can fit the data with

$$f = \text{erf } c(w - w_t)\,. \qquad (23)$$

Next, we want the average cross section. At a given depth x, we have a spectrum $\varphi(x,E)$ of protons and hence, for a given target, the average cross section there is

$$\bar{\sigma}(x) = \int_0^\infty \varphi(x,E)\sigma(E)dE \Big/ \int_0^\infty \varphi(x,E)dE\,.$$

The normalization assures us that we can obtain a good approximation to these cross sections by considering the flux at the surface *only*, i.e., the change in shape of the spectrum with depth is small, so that the mean cross sections are relatively independent of depth.

At the surface, we have assumed the spectrum

$$\varphi_p(0,E) = pE^{1/2}e^{-qw}\,.$$

This yields

$$\bar{\sigma} = \sigma_\infty e^{\theta^2} e^{-qw_t} \operatorname{erfc}\theta\,, \qquad (24)$$

where

$$\theta \equiv q/2c\,.$$

Note that given E_t and σ_∞, one experimental value suffices to fix c. When θ is large, we may use the asymptotic expansion for the error function and find that

$$\bar{\sigma} \cong \sigma_\infty e^{-qw_t}/\pi^{1/2}\theta\,. \qquad (25)$$

Because of the very high effective threshold for production off high-A nuclei, there is effectively no production from them at intermediate energies.

Next, let us look at the experimental light-element spallation cross sections.[15,16]

Generally, the multiplicity of fragments goes down rapidly with charge; workers[17] have shown that at high energy,

$$\frac{N(Z)}{N(Z+1)} \cong \text{constant} = a\,, \qquad (26)$$

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[14] J. M. Dickson and T. C. Randle, Proc. Phys. Soc. (London) **A64**, 902 (1951); L. Marquez and I. Perlman, Phys. Rev. **81**, 953 (1951); G. H. Coleman and H. A. Tewes, *ibid.* **99**, 288 (1955); G. Carleson, Acta Chem. Scand. **8**, 1697 (1955); E. Baker, G. Friedlander, and J. Hudis, Phys. Rev. **112**, 1319 (1958); G. Rudstam, E. Bruninx, and A. C. Pappas, *ibid.* **126**, 1852 (1962); M. Honda and D. Lal, *ibid.* **118**, 1618 (1960); J. R. Grover, *ibid.* **126**, 1540 (1962); I. Dostrovsky, Z. Fraenkel, and J. Hudis, *ibid.* **123**, 1452 (1961).

[15] D. A. Kellogg, University of California, Lawrence Radiation Laboratory Report UCRL-1899, 1952 (unpublished).

[16] I. Dostrovsky, Z. Fraenkel, and L. Winsberg, Phys. Rev. **118**, 781 (1960); I. Dostrovsky, Z. Fraenkel, and P. Rabinowitz, *ibid.* **118**, 791 (1960); L. E. Bailey, thesis, University of California, Lawrence Radiation Laboratory Report UCRL-3334, 1956 (unpublished).

[17] O. V. Lozhkin and N. A. Perfilov, Zh. Eksperim. i Teor. Fiz. **31**, 913 (1956); N. S. Ivanova, V. I. Ostroumov, and Yu. V. Pavlov, *ibid.* **37**, 1604 (1959); N. A. Perfilov, N. S. Ivanova, O. V. Lozhkin, M. M. Makarov, V. I. Ostroumov, Z. I. Solov'eva, and V. P. Shamov, *ibid.* **38**, 345 (1960); U. R. Arifkhanov, M. M. Makarov, N. A. Perfilov, and V. P. Shamov, *ibid.* **38**, 1115 (1960); O. V. Lozhkin, N. A. Perfilov, A. A. Rimskii Korsakov, and J. Fremlin, *ibid.* **38**, 1388 (1960); P. A. Gorichev, O. V. Lozhkin, and N. A. Perfilov, *ibid.* **41**, 35 (1961) [English transls., respectively: Soviet Phys—JETP **4**, 790 (1957); **10**, 1137 (1960); **11**, 250 (1960); **11**, 806 (1960); **11**, 1001 (1960); **14**, 27 (1962)].

a reasonable average for this constant being 1.3. Moreover, Eq. (26) is consistent with Rudstam's results.[12] The resulting ratios are in reasonable agreement with measurements made by Gradsztajn *et al.*[18] at 156 MeV with an O^{16} target. The best values[19] from their papers are

$$\begin{aligned}\sigma(\mathrm{Be}^7) &= 5.2\text{ mb},\\ \sigma(\mathrm{Li}^6) &= 9.8\text{ mb},\\ \sigma(\mathrm{Li}^7) &= 12.9\text{ mb}.\end{aligned} \tag{27}$$

These values are good to $\pm 20\%$. The value for Be^7 is consistent, within experimental error, with the value 4.2 mb obtained from Eq. (21) at 156 MeV.

Since Be^7 rapidly decays to Li^7, the total effective Li^7 production at 156 MeV off O^{16} is $\sigma(\mathrm{Li}^7+\mathrm{Be}^7)=18.1\pm3.5$ mb. For O^{16}, $\sigma_\infty \cong 12$ mb and Eq. (22) $\rightarrow E_t \sim 24$ MeV. If we use Eq. (23), then with the value given in (27),

$$q=0.003 \rightarrow \bar{\sigma}=0.69\text{ mb}.$$

Because of secondary production, we should increase these values by some small fraction—say 15%; thus,

$$\bar{\sigma}(\mathrm{Be}^7,\mathrm{O}^{16}) \cong 0.8\text{ mb}.$$

All the excitation functions are assumed to be similar to those for Be^7, but appropriately normalized. Thus for Li^7, the ratio is $12.9/5.2=2.5$. Hence,

$$\bar{\sigma}(\mathrm{Li}^7+\mathrm{Be}^7\ \mathrm{O}^{16})=0.8(1+2.5)=2.8\text{ mb}.$$

However, all calculations based on the resulting cross sections give $N(\mathrm{Li}^7)/N(\mathrm{B}^{10})$ consistently low. (See Figs. 10–13.) We find that we can achieve greater internal consistency by taking $a\cong 1.8$ rather than 1.3. Indeed, we would expect a to increase as the proton energy decreases. This also gives us cross sections very close to those obtained by adjusting *ad hoc* in order to obtain ratios consistent with the abundances. The values thus found are given in Table I, where the numbers in parentheses are the experimental values. Note that for these new values, $\sigma_s(\mathrm{B}^{11})/\sigma_s(\mathrm{Be}^9)=1.84$, which is approximately the observed terrestrial ratio. Hence, any large enrichment of B^{11} from $C^{12}(n,2n)$ must be accompanied by a proportionate increase in the Be^9 cross section. Because of the higher threshold, the contribution from the silicon group (and certainly from heavier elements) is negligible in comparison to production off oxygen and carbon for this low-energy spectrum.

TABLE I. Measured and calculated spallation cross sections off O^{16} at 150 MeV. Measured values are in parentheses.

Isotope	σ_s (mb)
Li^6	(9.8)
Li^7	(12.9+5.2=18.1)
Be^7	(5.2)
Be^9	2.5
Be^{10}	1.7
B^{10}	7.5
B^{11}	4.6

Neutron production. At proton energies in excess of 100 MeV, we find[16,20] that at each energy the multiplicity M_n of neutron production per proton is a linear function of N, the neutron numbers of the target:

$$M_n(E)=Nk(E).$$

$k(E)$ must increase from zero at threshold to unity as $E\rightarrow\infty$, corresponding to total breakup of the nucleus. If we ignore any possible resonances, the function must be monotonic with E. We find the total neutron production spectrum at low energies from copper, by summing the various excitation functions (p,xp,yn).[21] We find that the cross section can be well approximated by

$$\sigma_n(E)=g(E-E_t)^{1/2},$$

where E_t is the threshold energy. The same holds[22] for Ag^{109}. It is analytically more convenient for our calculations, however, to use

$$\sigma_n(E)=g(w-w_t)^{1/3}.$$

(Note that w_t for the given target here is usually smaller than for spallation. We will use w_t' for the neutron production threshold whenever confusion might arise.) From the Cu values, we then find

$$k(E)=0.0092(w-w_t)^{1/3}. \tag{28}$$

This expression increases without bound, but it will reach unity only for $E\sim 10$ GeV, while our (phenomenological) flux is already negligible in this neighborhood and falls far faster than $k(E)$ rises. Hence we make no appreciable error in using (28) to compute a mean value for σ_n. Since

$$\sigma_n(E)=\sigma_r M_n(E),$$

and assuming the reaction cross section σ_r is independent of energy, we find, by folding (28) into the proton spectrum at the surface,

$$\bar{\sigma}_n \cong 0.0092 N\sigma_r \Gamma(\tfrac{4}{3})q^{-1/3}e^{-qw_t}. \tag{29}$$

σ_r can be obtained either experimentally or by using the Serber expression (geometrical cross section times transparency):

$$\sigma_r=37A^{0.71}\text{ mb}. \tag{30}$$

Note that (29) and (30) together are consistent with the

[18] E. Gradsztajn, M. Epherre, and R. Bernas, Phys. Letters 4, 257 (1963). I am indebted to Professor Bernas for making his results available to me before publication.

[19] E. Gradsztajn, J. Phys. Radium 21, 761 (1960).

[20] W. E. Crandall and G. P. Millburn, University of California, Lawrence Radiation Laboratory Report UCRL-2706, 1954 (unpublished); C. N. Waddell, T. M. Henderson, and P. S. Lewis, in *Proceedings of the Rutherford Jubilee International Conference, Manchester, 1961*, edited by J. B. Birks (Academic Press Inc., New York, 1961), p. 171; E. L. Fireman and F. S. Rowland, Phys. Rev. 97, 780 (1955).

[21] J. W. Meadows, Phys. Rev. 91, 885 (1953).

[22] J. Wing and J. R. Huizenga, Phys. Rev. 128, 280 (1962).

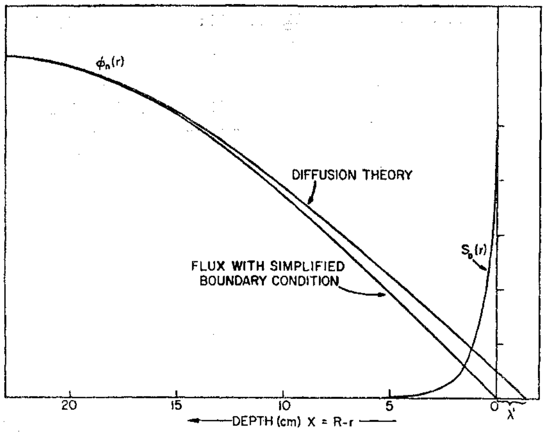

FIG. 2. Neutron flux φ_n and spallation production $S_a(r)$ near the surface of the sphere, in arbitrary units. The extrapolation distance λ' is shown explicitly ($R=300$ cm).

expression

$$\sigma_n(A)\cong k'A^{1.65}$$

found by Gross[23] in his 190-MeV experiments.[24] The effective reaction and neutron production cross sections for $q=0.003$ are given in Table II.

It is quite possible that the primitive flare proton spectrum was harder, as suggested by Fowler.[3,25] This corresponds to a decrease in q. Consider how the various quantities change with q. From Eqs. (15), (16), (18), (25), and (29), we find (with $\lambda\gg\alpha/q$) that decreasing q increases the skin depth, the flux, and the effective spallation cross sections. We can then keep neutron or spallation (but not both) production constant by lowering p. Finally, we have not considered secondary neutron production here, which might be quite significant. This effect will be treated more adequately in a subsequent paper.

IV. NEUTRON ABSORPTION

In order to calculate the neutron flux and resulting absorption, we use Fermi age theory.[26] Even though it is not, strictly speaking, valid for hydrogenous media, it will give us results that are adequate within the approximations made here.

When there is no absorption, the (differential) neutron flux between Fermi ages τ and $\tau+d\tau$ is just

$$d\varphi_n(r,\tau)=q(r,\tau)|d\tau|,$$

where the slowing-down density

$$q(r,\tau)=\sum_{n=1}^{\infty}A_n\left(\frac{\sin B_n r}{r}\right)\exp(-B_n^2\tau)$$

is the solution of the Fermi "age equation" for neutrons,

$$\nabla^2 q=\partial q/\partial\tau,$$

in spherical geometry, with the boundary condition that the neutron flux must vanish at the "extrapolated" boundary,

$$a=R+\lambda'.$$

The boundary condition is satisfied for the eigenvalues

$$B_n=n\pi/a,$$

and

$$A_n=(2a/\pi^2)\int_0^\pi S_N(ax/\pi)x\sin nx dx, \qquad (31)$$

where $S_N(r)$ is the neutron source function, given by Eqs. (19) and (20), with σ_N replacing σ_a. λ' is of the order of 1 cm. For spheres even as large as $R=10$ cm, however, the neutron flux is so low that no appreciable error is made by assuming $\lambda'=0$. For larger spheres, λ' is only a small perturbation on R and the deviations of the object from sphericity will be at least as great. Hence we take $a\cong R$, which simplifies the integrals considerably. However, although this approximation will permit a good calculation of the neutron flux in the sphere, it is precisely at the boundary that we need a very good calculation of φ, since it is just in a thin skin at the surface that spallation occurs. As we see from Fig. 2, the more precise boundary condition leads to considerably more neutron irradiation of the spallation products. Calculation shows that the flux is linear to a depth of at least 6 cm, i.e.,

$$\varphi(r)\cong a(R-r)$$

near the surface. The "extrapolated" flux is indistinguishable from that vanishing at the surface, by the time the maximum is reached (at a depth of 15 cm or so). We therefore use the slope $a'=[15/(15+\lambda')]a$ instead. Moreover, the exact transport-theory calculation shows that there is a dip in the flux, within a mean free path or so of the surface, to approximately 81% of the asymptotic value. Thus we will do quite adequately if in the spallation calculations we approximate the neutron flux near the surface by

$$\varphi(r)\cong a'(R-r+\lambda')[1-0.1873e^{-(R-r)/\lambda_t}],$$

TABLE II. Effective neutron production and reaction cross sections, (in mb); $q=0.003$.

Element	H	C^{12}	O^{16}	Mg^{24}	Al^{27}	Si^{28}	Ca^{40}	Fe^{56}
$\bar{\sigma}_n$	0	55	104	215	289	278	531	1020
σ_r	45	210	259	343	373	382	490	620

[23] E. Gross, University of California Lawrence Radiation Laboratory Report No. UCRL-3330, 1956 (unpublished).

[24] I am indebted to Professor K. Strauch for the prepublication use of paper D-2, in *Proceedings of the Symposium on Protection against Radiation Hazards in Space*, TID 7652 (U. S. Atomic Energy Commission, Gatlinburg, Tennessee, 1962), p. 409.

[25] W. A. Fowler (private communication).

[26] S. Glasstone and M. C. Edlund, *The Elements of Nuclear Reactor Theory* (D. Van Nostrand Company, Inc., 1952).

where $\lambda_t = 3D$ is the transport mean free path and the extrapolation distance[27] is given by the expression

$$\lambda' \cong [(\Sigma_s + \Sigma_{ab})/\Sigma_s][0.7104 + 0.6229 e^{-1.156\sqrt{(R/\lambda_t)}}]\lambda_t .$$

While slowing down to thermal energy, neutrons are captured, the number of captures in the lethargy[24] interval du being $\bar{\varphi}(r,u)\bar{n}\sigma_a(u)du$ cm^{-3} sec^{-1}. Hence, in slowing down, the number captured by nuclei a is

$$C_a = \bar{n}\int_0^{u_t} \sigma_a(u)\,\bar{\varphi}(r,u)du \text{ cm}^{-3}\text{ sec}^{-1}, \qquad (32)$$

where $\bar{n}$ is the number density of nuclei. Assuming $1/v$ absorption of neutrons by nucleus a, the absorption cross section at lethargy u is

$$\sigma_a(u) = \sigma_{at} e^{(u-u_t)/2},$$

where u_t is the thermal lethargy. The scattering cross section also increases with u, but more slowly:

$$\Sigma_s = \Sigma_{st} e^{\epsilon(u-u_t)},$$

where $\epsilon < 0.1$. The lethargy and the Fermi age of the neutron are related by the expression

$$\tau(u) = \int_0^u \frac{Ddu}{\xi\Sigma_s + \gamma\Sigma_{ab}}, \quad \tau \leq \tau_t .$$

Assuming $\gamma\Sigma_{ab} \ll \xi\Sigma_s$, we find that

$$\tau(u) = \frac{D}{\xi\Sigma_s}\left(\frac{1-e^{-2\epsilon u}}{2\epsilon}\right)\exp(2\epsilon u_t^0), \qquad (33)$$

where D, Σ_s, and u_t^0 are the values at 20°C, ξ is the mean logarithmic energy decrement, Σ_s and Σ_{ab} the macroscopic neutron scattering and absorption cross sections, and γ is a parameter numerically comparable to ξ. The resonance escape probability is then given by the equation

$$\ln p(u) = -\frac{2[e^{u(0.5-\epsilon)} - 1]}{m(1-2\epsilon)} \exp[u_t^0(\epsilon - 0.5)],$$

where m is the moderating ratio:

$$m = \xi\Sigma_s/\Sigma_{ab} .$$

Inserting these expressions in (32), we can calculate C_a. This involves the integrals

$$\Sigma_a C_n = \int_0^{u_t} \frac{\Sigma_a(u)p(u)\exp(-B_n^2\tau)du}{\xi\Sigma_s(u) + \gamma\Sigma_{ab}(u)} .$$

We define:

$$J = (\xi\Sigma_s 2\epsilon/D)\exp(-2\epsilon u_t^0),$$
$$H = (\Sigma_{ab}/D)\exp[-u_t^0(\tfrac{1}{2}+\epsilon)],$$
$$\eta = 4\epsilon/(1-2\epsilon), \qquad f = H\eta/J,$$
$$g_n = B_n^2/J, \qquad K_n = (\Sigma_a/\Sigma_{ab})fe^{f-g_n} .$$

[27] A. M. Weinberg and E. P. Wigner, *The Physical Theory of Neutron Chain Reactors* (University of Chicago Press, Chicago, 1958).

Then, ignoring $\gamma\Sigma_{ab}$ as before, we find that

$$\Sigma_a C_n = K_n \int_1^{e^{2\epsilon u_t/\eta}} \exp(g_n x^{-\eta}) e^{-fx} dx .$$

To integrate this, we make the analytic approximation

$$\exp(gx^{-\eta}) \cong 1 + [(e^g - 1)/x^\nu] .$$

In order to have the approximation fit at two relaxation lengths, we set

$$\nu = \eta \ln(1 + e^{g/2})/\ln 2 .$$

Then writing the upper limit as

$$M = e^{2\epsilon} u_t/\eta ,$$

we have

$$\Sigma_a C_n = K_n\left[\frac{1}{f}(1-e^{-fM}) + \frac{(e^{g_n}-1)}{(1-\nu)}(M^{1-\nu}-1)\right].$$

The second term is really the first term of a series. When g_n is large, it is a good approximation. When g_n is small, the series converges slowly, but its absolute value is small compared to the first, and so the approximation is adequate.

Thus, finally, the absorption by nucleus a during slowing down can be written as the product of the macroscopic thermal-neutron absorption cross section Σ_a, and an effective fast-neutron flux:

$$C_a = \Sigma_a \varphi_F(r),$$

where

$$\varphi_F(r) = \sum_{n=1}^{\infty} A_n \frac{\sin B_n r}{r} C_n .$$

Although the neutrons will be created with a spectrum of energies, it will be adequate to assume, as Eberhardt *et al.*[11] do, that the neutrons are created at 3.68 MeV. This is because of the rapid slowing down of the neutrons, and is mathematically reflected in the fact that the initial neutron energy appears as the upper limit of the integrals in the calculation of φ_n, and can (except for the low-energy tail, which is, of course, small) be replaced by ∞, to a high degree of approximation. Hence,

$$u_t = \ln(E_{\text{initial}}/E_{\text{th}}) = \ln(2.86\times 10^{10}/T)$$

with T in °K, and $u_t^0 = 18.4$. Once the surviving neutrons have slowed down to (ambient) thermal energies, they diffuse through the sphere, the thermal flux φ_T satisfying the diffusion equation

$$D\nabla^2\varphi_T - \Sigma_{aT}\varphi + pq(r,\tau) = 0 .$$

As above, the solution is

$$\varphi_T(r) = \frac{p(u_t)}{D}\sum_{n=1}^{\infty}\frac{A_n e^{-B_n^2\tau}}{\kappa^2 + B_n^2}\frac{\sin B_n r}{r},$$

where

$$\kappa^2=\Sigma_{ab}/D.$$

Thus, the total absorption by nucleus a is

$$\Sigma_a(\varphi_F+\varphi_T),$$

and

$$\beta_a(r)=\sigma_{at}[\varphi_F(r)+\varphi_T(r)]\ \mathrm{sec}^{-1}.$$

Note that the values of the material parameters are at the *ambient* temperature T. τ is roughly independent of T, but

$$\kappa^2=\kappa_0^2\exp[(u_t-u_t^0)(\tfrac{1}{2}+\epsilon)],$$

where κ_0^2 is the value at 20°C.

Finally, we must evaluate A_n. If we neglect the extrapolation distance, Eq. (31) implies that

$$A_n=(S_N/2\pi nk)(-1)^n\{(2k^2R^2/n\pi)\tan^{-1}(n\pi/kR)+(kR/n\pi)^2\ln[1+(n\pi/kR)^2]-2kR-1+\epsilon_n\},$$

where the correction term

$$\epsilon_n\cong e^{-2kR}+\frac{2k^2R^2}{n\pi}\left\{\frac{Ei(-2kR)}{n\pi}+\frac{e^{-2kR}(1-e^{-2kR})}{3[(n\pi)^2+(kR)^2]}\left(\frac{kR}{n\pi}-n\pi\right)\right\}$$

is negligible for large kR.

The isotopes under consideration are H^2, Li^6, Li^7, Be^9, B^{10}, B^{11}. In the case of B^{10}, one of the principal contributors is Be^{10}. This is a long-lived parent, however, with half-life 2.7×10^6 yr. Thus, in the appreciable time before it decays to B^{10}, it is unaffected by the neutron flux it is exposed to. Hence, less B^{10} is transformed to Li^7 by $B^{10}(n,\alpha)Li^7$, and therefore the B^{10} should be enhanced, and the Li^7 suppressed (by the same amount). This correction term is found to be

$$C_{10}=4\pi\sigma_{\mathrm{Be}^s}\tau_m\int_0^R F(r)r^2\left[1-\frac{e^{-\beta_{10}(r)t}-\tau_m\beta_{10}(r)e^{-t/\tau_m}}{1-\tau_m\beta_{10}(r)}\right]dr,$$

where $\tau_m=3.9\times10^6$ yr is the mean life of Be^{10}, and $\sigma_{\mathrm{Be}}{}^s$ is the spallation cross section of Be^{10}. Then the total B^{10} produced is

$$\mathrm{B}^{10}=N_{10}+C_{10},$$

and

$$\mathrm{Li}^7=N_7+\mathrm{B}^{10}{}_s-\mathrm{B}^{10}=N_7(1+\sigma_{10}{}^s/\sigma_7{}^s)-\mathrm{B}^{10},$$

where $\sigma_a{}^s$ is the mean spallation cross section for nucleus a (including all contributors, e.g., C^{10} and Be^{10} as well as B^{10}, for B^{10}). $B^{10}{}_s$ is the B^{10} directly spallated, N_{10} the amount after neutron irradiation (but before correction for the Be^{10} effect). Finally, since the neutron absorption cross sections are negligible for Li^7, Be^9, and B^{11},

$$N_7=\mathrm{Li}^7{}_s,$$

etc. We want the abundances relative to $Si^{28}=10^6$. These relative abundances are given by the equation

$$\bar{N}_6=\mathrm{Li}^6(V_{\mathrm{Si}}/V),$$

etc., where V_{Si} is the volume occupied by 10^6 silicon atoms in the given mixture, and V is the volume of the sphere.

V. PRODUCTION OF DEUTERIUM

The deuterium concentration at r at time t is given by the expression

$$\frac{d\mathrm{D}(r,t)}{dt}=S_2(r)+S_2'(r)-L_2(r,t)-L_2'(r,t),\quad(34)$$

where S_2 is the production by spallation and $S_2'(r)$ is the production by the $p(n,\gamma)d$ process:

$$S_2'(r)=\beta_2(r)n_{\mathrm{H}}(r),$$

while $L_2(r,t)$ is the loss by diffusion out of the volume element, and L_2' the loss by $p+d\rightarrow 2p+n+\gamma$. n_{H} is the hydrogen concentration. The ratio of deuterium produced in the skin by spallation, when we take the disintegration probability into account, to that produced when we ignore it, is

$$\mathrm{D}_s'/\mathrm{D}_s\cong[1-\exp(-\sigma_a\varphi_0 t/4)]/\sigma_a\varphi_0 t/4.$$

σ_a, the $d(p,n+2p)\gamma$ cross section, is of the order of 1 b.[28] We shall see that $\varphi_0\sim10^{10}$ p/cm² sec, so that in 10^7 yr, $\mathrm{D}_s'/\mathrm{D}_s\cong0.63$, and hence, most of the spallated deuterium survives. Both the injected protons and the spallated deuterium, when stopped, pick up an electron and become highly reactive atomic hydrogen. Thus, the skin is subjected to an intensely reducing environment. Very little of the hydrogen will combine into H_2 and diffuse towards the center or leak out of the sphere; most of it will become bound to the oxygen in the matrix material.[29]

The range of a deuteron created by the $p(n,d)\gamma$ process, due to photon recoil, is only about 6 Å. Hence it can be recaptured before leaving the ice crystal in which it is formed. If there is any free (light) hydrogen in the area, the probability of the deuteron being recaptured will be approximately $p_r\cong r/(1+r)$, where r is the local ratio of free deuterium to free hydrogen, including the newly created deuterium. With little free hydrogen around, r is large, and p_r close to unity. Hence, the L_2 term will be very small. We take the L_2' term into account in an approximate way by multiplying the contribution to D/H due to spallation alone, calculated assuming $L_2=0$, by $\mathrm{D}_s'/\mathrm{D}_s$.

Thus, dropping the loss terms, we can immediately integrate Eq. (34) over the volume and with respect to time. For small spheres, the injected hydrogen is

[28] J. L. Friedes and M. K. Brussel, Phys. Rev. **131**, 1194 (1963).
[29] G. Allen (private communication).

approximately homogeneously distributed throughout the volume, whereas for large objects, the protons are stopped in the skin, and (to a first approximation) at the mean depth $\alpha\bar{E}^{1.5}$. The mean energy $\bar{E}$ is given by Eq. (15′). Hence, this mean depth is $\Delta=1.84\alpha/q$. Then dividing by the total hydrogen in the object after synthesis, the deuterium/hydrogen ratio is given by

$$\frac{\mathrm{D}}{\mathrm{H}}=\frac{\mathrm{D}_s'}{\mathrm{D}_s}\frac{S_2t}{N_{\mathrm{H}}}+\frac{n_0t}{N_{\mathrm{H}}}\int\beta_2dV+\frac{\dot{N}_{\mathrm{H}}}{N_{\mathrm{H}}}\frac{t^2}{2}\times\begin{cases}\beta_2(R-\Delta), & R>\Delta,\\ (1/V)\int\beta_2dV, & R<\Delta.\end{cases}\tag{34'}$$

$$S_2=\frac{\pi R^2}{k}\sigma_dn\varphi_p(1-f)\ \text{deut/sec},$$

where σ_d is the spallation cross section for deuterium. σ_d is estimated from the calculations of Dostrovsky *et al.*,[13] and $\sigma_d\cong\sigma_n/14$. Finally,

$$f=(2/\xi^2)[1-(1+\xi)e^{-\xi}]$$

is the transparency of the sphere (where $\xi\equiv2kR$). n_0 is the number density of the initial hydrogen, which is assumed to be homogeneously distributed throughout the sphere. The last term in (34′) is the contribution to the $n(p,d)\gamma$ reaction from the injected protons themselves. $\dot{N}_{\mathrm{H}}$ is the injection rate, i.e., the total number of protons stopped in the sphere per second, and not reacting. It is given by

$$\dot{N}_{\mathrm{H}}=\pi R^2\varphi_p(q/\alpha k)(1-f)=\pi R^2(2p/3\alpha k)(1-f).$$

Finally, N_{H} is the total (final) hydrogen in the sphere,

$$N_{\mathrm{H}}=n_0V+\dot{N}_{\mathrm{H}}t.$$

Even if considerable amounts of ice are subsequently lost by evaporation or other processes, and since hydrogen diffuses by migration[30] of H_2O, then aside from a possible small numerical factor due to the difference of diffusivity of H_2O and HDO because of their different masses, the ratio will remain unchanged.

VI. DESTRUCTION OF NUCLEI

When there is a nonvolatile isotope X already present, whose nucleus has a large neutron capture cross section, such as Gd^{157}, it will be partly destroyed in the regions where there is an appreciable thermal neutron flux, so that at time t,

$$X(r,t)=X(r,0)e^{-\beta_x(r)t}.$$

If the initial distribution is uniform, as we assume here, then the fraction remaining in the sphere after time t is

$$\frac{X(t)}{X(0)}=\frac{\int X(r,t)dV}{X(0,0)\frac{4}{3}\pi R^3}=\frac{3}{R^3}\int_0^R r^2e^{-\beta_x(r)t}dr.$$

When σ_x is very large, reasonable proton fluxes may give rise to a β_x so large that the integrand becomes vanishingly small except near the skin and in the deep interior. In such a case, we can make a simple approximation for large spheres:

$$\frac{X(t,R)}{X(0,R)}\cong\left(\frac{R-\Delta}{R}\right)^3,\quad R>\Delta,\tag{35}$$

where Δ is the thickness of the shell within which there is essentially complete destruction. Typically, Δ will be of the order of one meter.

VII. BULK PARAMETERS

Because of the $1/v$ dependence of the reaction cross sections, there are two effects. First, the thermal neutrons having approximately a Maxwellian distribution, the *mean* reaction cross sections are those usually quoted as thermal, i.e., for $E=kT$ ($\approx1/40$ eV at room temperature), multiplied by $\pi^{1/2}/2$. Secondly, the value at temperature T must be

$$\sigma(T)=\sigma(20°\mathrm{C})(T_0/T)^{1/2}=\sigma(20°\mathrm{C})(293/T)^{1/2}.$$

Combining these two effects, we obtain the absorption cross section at T

$$\sigma_a(T)=\sigma_{at}(230/T)^{1/2},$$

where σ_{at} is the quoted thermal cross section, and T the temperature of the body.

The Fermi age of a neutron slowed down to energy E in a medium is given by Eq. (33), where u is the lethargy. When $2\epsilon u_t^0\ll1$, this simplifies to

$$\tau(E)=(D/\xi\Sigma_s)\ln(E_0/E),\tag{36}$$

where E_0 is its initial energy. For a mixture of compounds the age is given approximately by the equation

$$1/\tau^{1/2}=\Sigma_i(v_i/\tau_i^{1/2}),$$

where v_i is the volume fraction of the compound. The Fermi age is not given explicitly for most substances. However, we find[31] that

$$\tau(\mathrm{Fe})=180\ \mathrm{cm}^2,$$

and from Hughes[32] we find that $\epsilon=0$ for O, Mg, Al, Si, and Ca. This allows us to use Eq. (36), and we find the values listed in column 5 of Table V, with $u_t=18.4$.

[30] W. Kuhn and M. Thürkauf, Helv. Chim. Acta **41**, 938 (1958).

[31] E. Amaldi, in *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1959), Vol. 38/2, p. 1.

[32] D. J. Hughes and R. B. Schwartz, Brookhaven National Laboratory Report BNL-325, 2nd. ed., 1958 (unpublished).

TABLE III. Composition of the various mixtures considered in the text.

Compound i	FGH 1	FGH 2	Terrestrial	
H_2O (ice)	4	0.8	0,[a]	0.24[b]
$MgSiO_3$	1	1	1	
$CaAl_2O_4$	0.16	0.16	0	
Fe_2O_3	0.08	0.08	0.15	
Fe	0	0	1.20	

[a] Dry. [b] Hydrated.

A very good fit can be obtained[7] for the range-energy curve for protons in materials between 50 and 500 MeV by using the relation

$$R\rho=5.3\times10^{-5}(110+Z_e)E^{1.6},$$

where the range R is in cm, ρ is in g/cm^3, and E in MeV. If we use $E^{1.5}$ instead, then with $R=\alpha E^{1.5}$, we find that

$$\alpha\cong8.8\times10^{-5}(110+Z_e)/\rho,$$

where Z_e is the mean atomic number. The mean neutron production is given by

$$\bar{\sigma}_n=\sum_\mu a_\mu\bar{\sigma}_{n\mu},$$

where a_μ is the abundance of element μ.

As an example, let us take the elemental abundances suggested by FGH for the "intermediate stage," with the oxygen abundance increased so as to permit oxidation of all the metals:

$$N(\mathrm{H})\sim8,$$
$$N(\mathrm{O})\sim8,$$
$$N(\text{Si group})\sim2.5,$$
$$N(\text{Fe group})\sim0.15.$$

(These are normalized to $N(\mathrm{Si})=1$.) We call this "FGH mixture 1." This corresponds, in their calculations, to a 10:1 shielding factor. We easily find reasonable compounds to satisfy these abundances, and these are displayed in Table III. The relevant atomic and nuclear parameters are given in Table IV. The values in Table V are then derived, except for H_2O, which is strongly anomalous, and for which experimental values

TABLE IV. Atomic and nuclear parameters for relevant elements.

Element	Z	A	$1-\frac{2}{3A}$	ξ	Thermal scattering and absorption cross sections in barns σ_s	σ_a
H	1	2	0.333	1.000	20.38	0.332
O	8	16	0.958	0.120	4.24	0.
Mg	12	24	0.972	0.0811	3.7	0.061 (4)
Al	13	27	0.975	0.0723	1.51	0.215 (8)
Si	14	28	0.976	0.0698	2.4	0.13
Ca	20	40	0.983	0.0492	3.1	0.43
Fe	26	56	0.988	0.0353	11.80	2.53

TABLE V. Neutron-scattering parameters for FGH mixture 1.

Compound i	$n_i\times10^{-22}$	D^{-1} (cm)	$\xi\Sigma_s$ (cm^{-1})	τ_i (cm^2)	v_i
H_2O (ice)	3.07	6.46	1.40	38	0.664
$MgSiO_3$	1.98	1.07	0.0394	435	0.258
$CaAl_2O_4$	1.40	0.934	0.0337	585	0.058
Fe_2O_3	1.97	2.10	0.0466	188	0.020
Mixture	2.67	4.66	0.944	65.1	1.000

are used. D is the neutron diffusion coefficient, $\xi\Sigma_s$ the slowing-down power, τ the Fermi age, and v_i the volume fraction of compound i. The resulting bulk parameters for this mixture are shown in column 2 of Table VI. These are calculated (except for ice) at 20°C and for $q=0.003$.

When we note that Gd^{157} has the huge thermal neutron-absorption cross section $\sigma=2.4\times10^5$ b, it is legitimate to ask whether the rare earths will not materially augment Σ_a.

On an abundance scale where $\log_{10}N(\mathrm{Mg})=7.40$, $\log_{10}N(\mathrm{Gd}^{157})=0.25$; hence, the relative abundance in this mixture would be

$$\frac{1.00}{18.52}\times\frac{10^{0.25}}{10^{7.40}}=3.82\times10^{-9},$$

and the concentration would therefore be

$$n(\mathrm{Gd}^{157})=3.82\times10^{-9}n_a=3.60\times10^{14}\ \mathrm{cm}^{-3}.$$

Finally, with the above cross section, we have

$$n\sigma_{\mathrm{th}}(\mathrm{Gd}^{157})=8.65\times10^{-5}\ \mathrm{cm}^{-1}.$$

If we double this to roughly take into account the rest of the rare earths, we still have only 1% of the macroscopic absorption cross section of the rest of the mixture, and we may therefore ignore the rare earths.

Finally, we come to the spallation cross sections. For FGH mixture 1, the oxygen contribution is 7.88/18.52 of the total, and the other target nuclei do not contribute appreciably, as we have seen. Therefore, we need merely multiply the numbers in Table I by 7.88/18.52. Next, consider FGH mixture 2. This corresponds[25] to a shielding factor of 50, rather than 10. We approximate this by taking only 20% as much H, and this is easily accomplished by taking $N(H_20)=0.8$, rather than 4.0. Next, we consider a "terrestrial" composition. Aller's values[33] for the earth are given in Table VII. If we normalize to Si$=1$, the number abundances are those given in column three. We can approximate these abundances with just pyroxene, hematite, and iron, in the relative abundances $N(MgSiO_3)=1.0$, $N(Fe_2O_3)=0.15$, and $N(\mathrm{Fe})=1.20$, as shown in Table III.

[33] L. H. Aller, *The Abundance of the Elements* (Interscience Publishers, Inc., New York, 1961), p. 35.

TABLE VI. Bulk parameters for the various mixtures, with $q=0.003$.

		FGH No. 1	FGH No. 2	Terrestrial (dry)
Density	ρ (gm/cm^3)	1.78	2.75	4.38
Mean Atomic No.	Z_e	5.86	8.58	13.32
	α	0.00574	0.00380	0.00248
Diffusion coeff.	D (cm)	0.215	0.381	0.634
Fermi age	τ (cm^2)	65.1	154	322
Neutron production	σ_n (mb)	89.3	148	343
Scattering cross section	$\bar{\sigma}_s$ (b)	11.1	6.89	5.53
Hydrogen density	n_{H} (cm^{-3})	4.07×10^{22}	1.74×10^{22}	0.
Atomic density	n_a	9.43×10^{22}	9.68×10^{22}	9.58×10^{22}
	V_{si} (cm^3)	1.96×10^{-16}	9.22×10^{-17}	7.25×10^{-17}
	Σ_s (cm^{-1})	1.05	0.666	0.530
	Σ_a (cm^{-1})	0.0173	0.0138	0.0550
Mean free path of proton	λ_p (cm)	57.4	40	28.5
	$\xi\Sigma_s$ (cm^{-1})	0.944	0.425	0.0394

VIII. RESULTS OF CALCULATIONS

Neutron fluxes. For spheres of icy silicate No. 1, the thermal and fast fluxes $\varphi(r,R)$ are shown in Figs. 3 and 4, respectively. They give φ as a function of radius, for various-sized spheres. The fluxes $\varphi(0,R)$ at the center of the spheres are shown in Fig. 5. The bump in the $\varphi_F(0,R)$ curve for small radii is inexplicable on physical grounds; it probably arises from the analytic approximations made in the program and is of no practical importance, as the total effective flux is very small for small radii in any case.

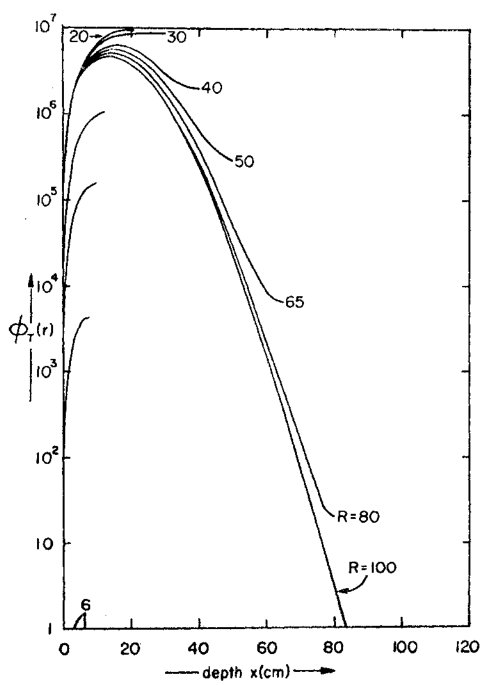

FIG. 3. Thermal neutron flux as a function of r, for spheres of icy silicate No. 1 composition, and radii R varying between 6 and 100 cm. $\chi=R-r$ is the depth into the sphere. $r=0$ where the curves terminate (i.e., at the center). The numerical labels for each curve indicate the radius of the corresponding sphere. $q=0.003$, $p=3\times10^7$, $T=170°$K.

Figures 6–8 show the results of the identical irradiation of terrestrial material. We see that the thermal fluxes have gone down by almost three orders of magnitude, because the terrestrial material is a very poor

TABLE VII. Terrestrial composition according to Aller.

Element	% by weight	Relative numerical abundance
Fe, Co, Ni	41.70	1.50
O	27.17	3.44
Si	13.84	1.00
Mg, S, Al, Ca	16.13	1.17
other	1.16	0.10

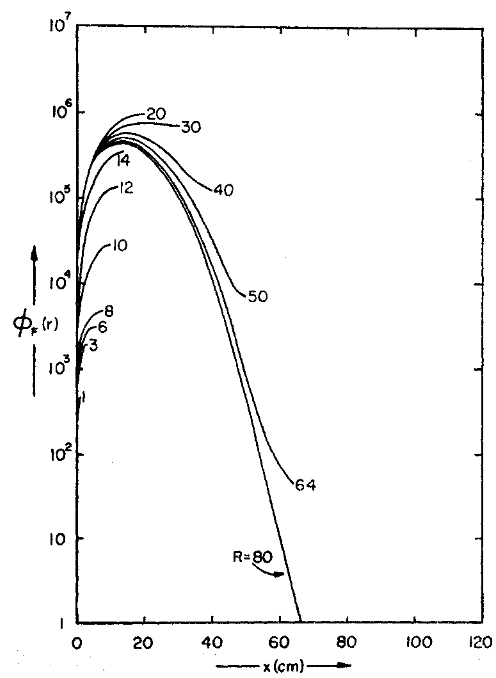

FIG. 4. "Fast "neutron flux $\varphi_F(r,R)$, as a function of r, for icy silicate No. 1 spheres. The labels are the same as in Fig. 3.

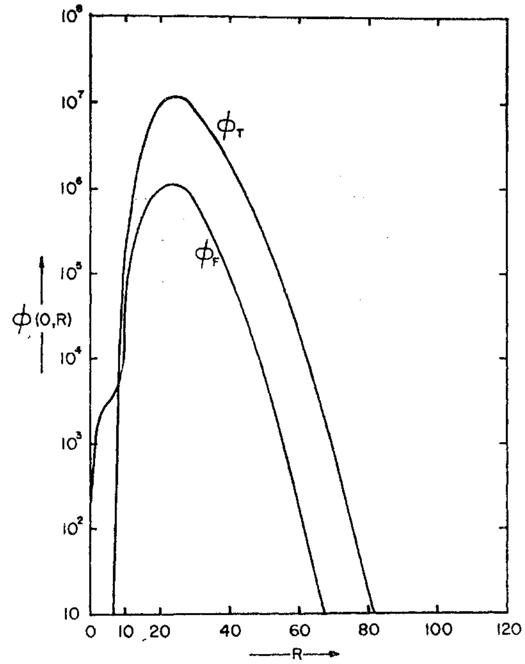

FIG. 5. Central values of "fast" and thermal neutron fluxes, as a function of radius R, for icy silicate No. 1 spheres.

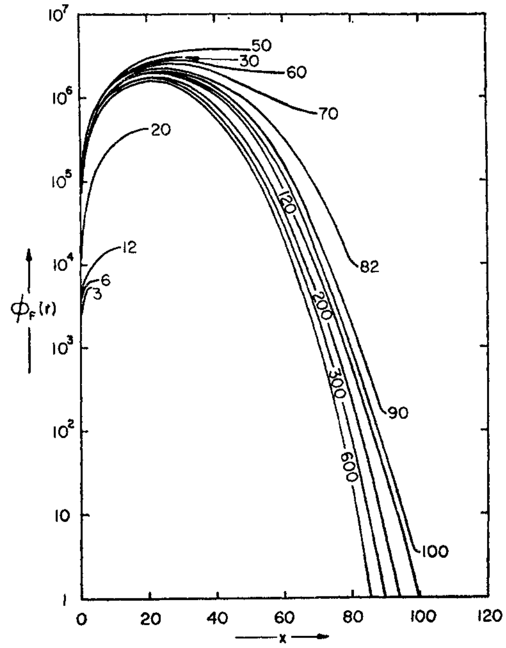

FIG. 7. Fast-neutron flux as a function of r, for spheres of dry terrestrial composition and varying radii R. $q=0.003$, $p=3\times10^7$, $T=170$°K. χ and the numerical labels on the curves have the same meanings as in Fig. 3.

moderator, compared to ice. On the other hand, the fast flux has actually increased, because there is proportionately more iron in the terrestrial mixture, and iron is a fairly strong neutron absorber.

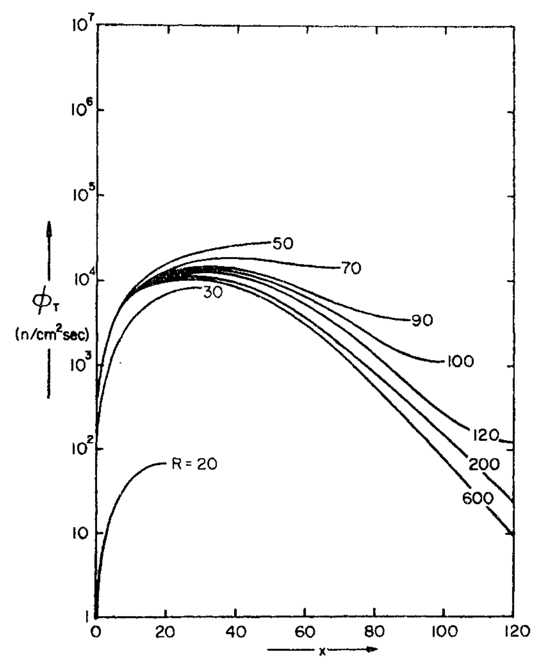

FIG. 6. Spatial variation of thermal neutron flux $\varphi_T(r)$ within various-sized spheres (of radius R) of dry terrestrial composition. χ and the numerical labels on the curves have the same meaning as in Fig. 3.

Next, consider the normalized light-element production, $\bar{N}_a$. In Fig. 9 $R\bar{N}_a$ are plotted versus R, for spheres of terrestrial composition. For $R>30$ cm or so, $R\bar{N}_a \propto N_a/R^2$ is constant. This is so because production is confined to a relatively thin "skin." B^{11} and Be^9 are unaffected by neutrons; hence, $R\bar{N}_{11}$ and $R\bar{N}_9$ are monotonic functions of R, and linear with p. In fact, for very small spheres, $F(r)\cong 1$; i.e., the proton irradiation will be all the way through, while the neutron irradiation will be negligible. Hence production must be proportional to the volume, and as $R\rightarrow 0$, $\bar{N}_a(R)\rightarrow$

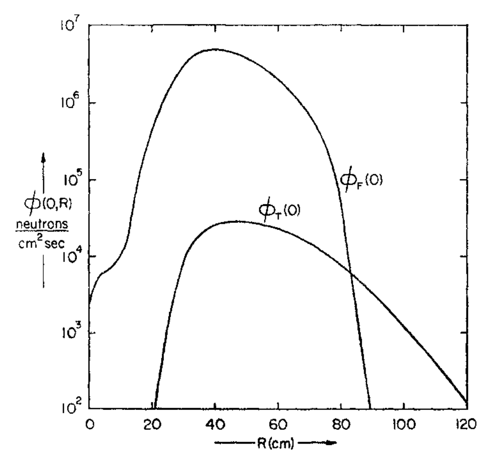

FIG. 8. Central values of thermal and fast neutron fluxes, as a function of radius, for spheres of dry terrestrial composition and varying radii. $q=0.003$, $p=3\times10^7$, $T=170$°K.

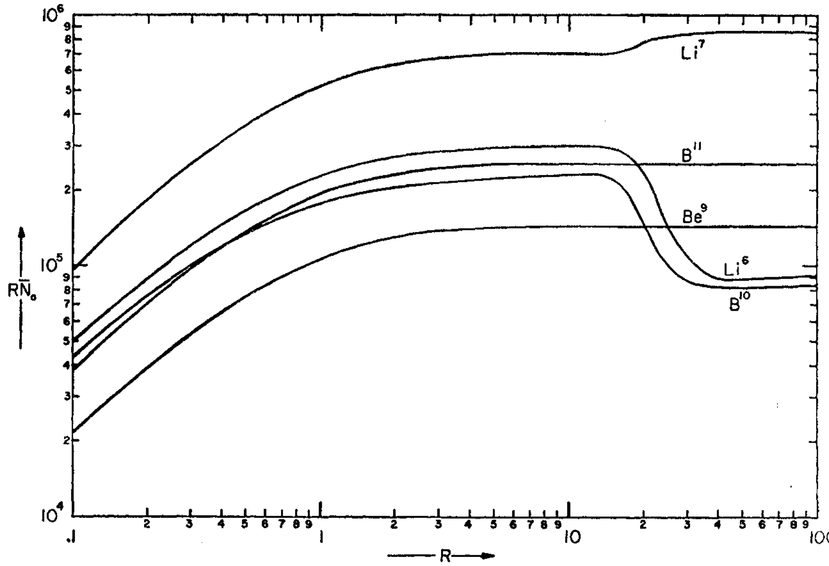

FIG. 9. Isotope abundances spallated from spheres of terrestrial composition. The irradiation is with $q=0.003$, $p=1.6\times10^9$, for $t=10^7$ years. The temperature is assumed to be $T=170°$K. $\bar{N}_a$ is the mean concentration of isotope X^a, normalized to $\bar{N}(\text{Si})=10^6$. Note that for large R, $R\bar{N}_a$ asymptotically goes to a constant, for all a. Hence the concentration $\bar{N}_a$ falls as $1/R$. This is because the production is a surface phenomenon, while the averaging is done over the whole volume.

const asymptotically. Thus, $R\bar{N}_{11}$ will be proportional to R, as is seen in Fig. 9. When the sphere becomes appreciable in size ($R>3k^{-1}$), only the skin is irradiated, and the production becomes proportional to the area, so that $R\bar{N}_{11}$ approaches a constant, as the figure shows.

Consider $R\bar{N}_{10}$. This behaves in the same way as $R\bar{N}_{11}$ up to $R\sim10$ cm, then drops precipitously before approaching a new asymptotic value. This is due to the neutron flux which builds up quite rapidly from negligible values for $R<10$ cm, to a maximum value at ~20 cm, beyond which the effect is constant. Precisely the mirror effect, of course, is found for $R\bar{N}_7$, since Li^7 is fed by the very reaction which depletes B^{10}. The deviations from linearity with p, due to the neutrons, are best seen by plotting the isotopic ratios as a function of p. This is done, for example, in Fig. 10. We see that the N_{11}/N_{10} and N_7/N_{10} ratios approach constants asymptotically, so that increasing the intensity of proton bombardment will have no effect on these ratios thereafter.

We next must compare the calculated abundances with those observed. Relative to Si$=10^6$, the observed abundances of the light elements[33–35] are given in Table VIII. The meteoritic value of Be has recently been revised downward[36] by a factor of about 30 from

TABLE VIII. Terrestrial and meteoritic (chondritic) light-isotope abundances.

	Isotope abundances (Si$=10^6$)				
	Li^6	Li^7	Be^9	B^{10}	B^{11}
Terrestrial	33±6	400±70	33±13	15±3	61±15
Meteoritic	3.1	33	3.4 (0.64)	1.6	6.1
	Isotope ratios				
	Li^7/Li^6	B^{11}/Be^9		B^{11}/B^{10}	Li^7/B^{10}
Terrestrial	12.0 ±0.2	1.85±0.75		4.1 ±0.1	27±7
Meteoritic	10.5[a]±0.2	1.8 ±0.6	(9.5±2.4)	3.85±0.05	21
	Crustal/meteoritic abundance ratio				
	Li		Be		B
	12±3		10±3 (52±17)		10±2

[a] A more recent measurement of the lithium isotope ratios, by O. Müller and D. Krankowsky (to be published), gives 12.17±0.11 for the meteoritic ratio, and they show that the meteoritic and terrestrial ratios agree to within 2% or better. If we then recalculate the solar energy output as is done at the end of Sec. IX, we obtain an unreasonably large value. We might conjecture that the protoplanetary gas disk grows thinner as we go further from the solar condensation, at just such a rate that the magnetic field lines focus the protons onto the protoasteroidal belt. To focus *just enough* to exactly compensate for the loss of protons (by absorption in the prototerrestrial planetesimals) is a rather artificial, *ad hoc* postulate, however. The similarity in the various ratios, therefore, remains a rather puzzling fact.

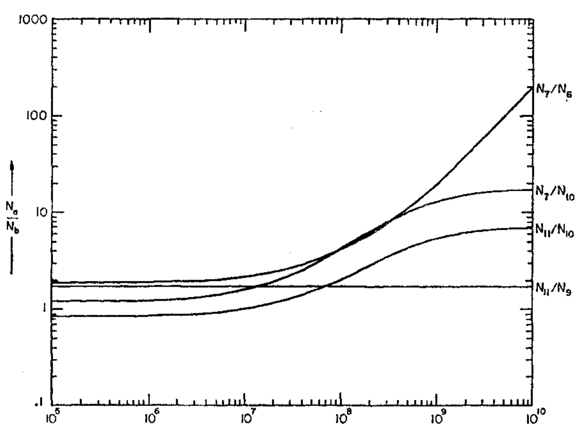

FIG. 10. Isotopic ratios as a function of irradiation intensity, for an 80-cm sphere of icy silicate No. 1 composition. T is assumed to be 170°K.

[34] M. Shima and M. Honda, J. Geophys. Res. 68, 2849 (1963).
[35] M. Shima, J. Geophys. Res. 67, 4521 (1962).

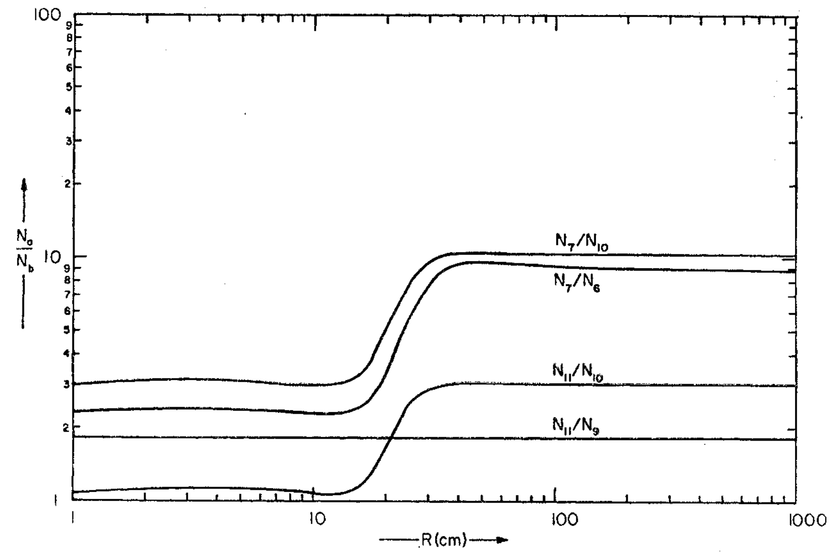

FIG. 11. The variation of the isotopic ratios with R, for terrestrial spheres. This is from the same calculation as Fig. 9. There is a very broad maximum at 50 cm. Calculations in the text have been made for 80-cm spheres, but the difference is completely insignificant.

the Suess-Urey (1956) value.[37] This is the only datum which does not seem consistent with the other data, and a value approximately five times as large has been adopted here in order to bring it into consistency with the others and with the solar value, in spite of the accuracy of their method. (The new measurement and the ratios obtained with it are displayed in parentheses.) Table VIII shows that the terrestrial and meteoritic isotope ratios are approximately equal, while the absolute terrestrial (crustal) abundances are approximately ten times larger than the meteoritic ones. Very likely these lithophile elements have been concentrated in the crust, perhaps by an order of magnitude. We shall return to this later. The ratios are much better known than the absolute abundances, especially for the earth. In the following, we have tried to fit the meteoritic isotope ratios.

In Fig. 11 we plot the ratios of the calculated abundances as a function of R for a given p. We see that these ratios attain a broad maximum for $R\sim 50$ cm. Hence, unless we take a value of p such that the N_7/N_{10} ratio is sufficiently large for $R=50$ cm, we will not be able to achieve the right ratio with *any* distribution of sizes. (In our calculations, we use $R=80$ cm, which makes no significant difference.)

Let us consider the FGH No. 2 mixture first. We note that since Be^9 and B^{11} both have negligible neutron absorption cross sections and are not fed by any chain of reactions, their abundances must be in the ratio of their spallation cross sections. The observed ratio is about 1.8, whereas the ratio of spallation cross sections first estimated (with $a=1.3$) is 1.74. The calculated ratios as a function of p are shown in Fig. 12. The best single determiner of the proper flux is the N_7/N_6 ratio, since this is, to first approximation, independent of σ_{10}, and the spallation cross section ratio is well known experimentally. From Fig. 12 we see that the observed N_7/N_6 ratio is achieved for $p=1.5\times 10^9$. However, consider the energies involved: The total output of protons during the period of synthesis was $A\varphi_p t$, where

$$\varphi_p=2p/3q=3.3\times 10^{11}\text{ protons/cm}^2\text{ sec},$$

and A is the effective area. This flux is about 3×10^4 times that required by FGH, and is due to the softness of our spectrum. With a primitive solar radius[3] of 3×10^{12} cm, and assuming the protons are emitted from the whole solar surface,

$$A\sim 10^{26}\text{ cm}^2,$$

then the total hydrogen output would have been at least

$$O_{\mathrm{H}}\sim 10^{52}\text{ hydrogen atoms}=2\times 10^{28}\text{ g}.$$

This is only one part in 10^5 of the total present mass of the sun, and hence does not seem unreasonable. On the other hand, the energy emitted would have been at least

$$O_E=At\varphi_E=Atpq^{-5/3} \quad (37)$$

which, with the p and q used above, becomes

$$O_E\sim 8.5\times 10^{53}\text{ MeV}\cong 1.4\times 10^{48}\text{ erg},$$

or 1.4×10^{41} erg/yr. This is a very high value, about equal to the *total* present-day energy output rate, and three orders of magnitude greater than the 10^{45} erg hypothesized by Hoyle.[38]

On the other hand, if small spheres (of radii >30 cm) give us the required ratios, then the absolute concen-

[36] C. W. Sill and C. P. Willis, Geo. et Cosmochim. Acta 26, 1209 (1962).
[37] H. E. Suess and H. C. Urey, Rev. Mod. Phys. 28, 53 (1956).
[38] F. Hoyle, Quart. J. Roy. Astron. Soc. 1, 28 (1960).

trations can be obtained by mixing these with unirradiated material (or with large spheres which are, essentially, unirradiated). This picture is consistent with that suggested by Whipple,[39] of a large, already solid protoearth being bombarded by irradiated comets. This hypothesis is particularly attractive, for these primitive "cometesimals" could have received very heavy proton irradiations if their perihelia were small.

There is a further difficulty: even if all the injected hydrogen is lost by diffusion, its kinetic energy is retained by the sphere and must raise its temperature. The hydrogen injection rate is given by the equation

$$\dot{N}_{\mathrm{H}}=\pi R^2 2p(1-f)/3\alpha k\,,$$

and the mean energy per proton is given by Eq. (15′). Thus the energy input rate is

$$\dot{U}=\frac{p(1-f)}{4\alpha k q^{2/3}}\ \mathrm{MeV/cm^2\ sec.}$$

For spheres of any appreciable size, $f\cong 0$, and hence for the icy silicate No. 2 spheres, and $p=1.5\times 10^9$, we have

$$\dot{U}\cong 10^7\ \mathrm{erg/cm^2\ sec}=0.22\ \mathrm{cal/cm^2\ sec.}$$

In order to radiate this away as fast as it comes in, the surface would have to be at a temperature T such that $\sigma T^4=\dot{U}$, that is, $T\cong 660$°K. This is too hot to maintain ice or even water. Of course, some of the heat will flow into the body; if all the energy went into heating the body, the energy input rate would be $\dot{W}=(3/R)\dot{U}$ MeV/cm³ sec. With a specific heat c_p cal/°K, we would have a temperature increase of $3\times 0.22/Rc_p$ °K/sec throughout the sphere. Even with $R=1$ km and $c_p=1$, this yields 200°K/yr. Thus even a sizable body will be heated to the black-body equilibrium temperature within a few years at most, volatilizing all the ice and leaving a hot, dry body.

Consider the possibility of a different composition; e.g., icy silicate No. 1. The results are shown (still with

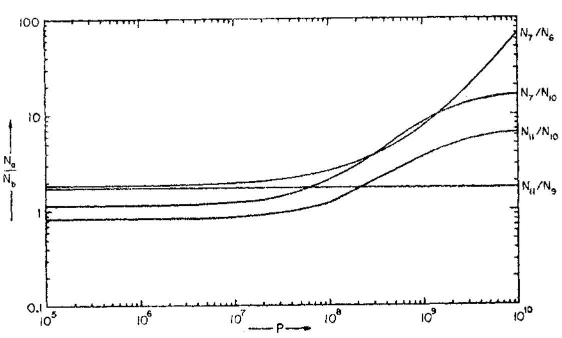

FIG. 12. Isotopic ratios as a function of irradiation intensity, for an 80-cm sphere of icy silicate No. 2 composition. T is assumed to be 170°K.

[39] F. L. Whipple (private communication).

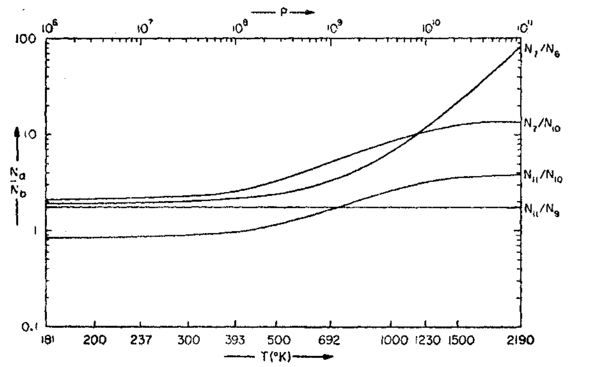

FIG. 13. Isotopic ratios as a function of irradiation intensity, for an 80-cm sphere of terrestrial composition. The correct temperatures are used and indicated below.

$a=1.3$) in Fig. 10. Proceeding as before, we find that the required intensity is only a factor 5 smaller than for the first case. This still corresponds to an equilibrium temperature of $T=490$°K.

Thus, the icy silicate will not be stable. We must therefore try to see whether we can obtain consistent ratios for a dry silicate body. We use the terrestrial composition given in Table VII. If we again assume $T=170$°K, the resulting curves are almost identical with those of Fig. 10. However, we must now use the appropriate temperature for each p. The black-body temperature from proton irradiation alone, with this value of q, is $T^4=230p$. If thermal radiation alone brings the temperature to (say) 170°K, then

$$T^4=230p+(170)^4.$$

The resulting isotopic ratios as a function of p, for an 80-cm "terrestrial" sphere, are shown in Fig. 13. The temperatures corresponding to various irradiation intensities are shown at the bottom of the graph. As we see from this figure, the curves are now much flattened, because as p increases, so does the temperature, and hence the thermal absorption cross sections decrease, so that the effect of the neutrons becomes less and less important. Still, we could obtain the desired ratios with $p\sim 10^{10}$. This, however, would yield the value $T>1230$°K, and the author knows of no hydrogen compound that will not decompose or volatilize at such temperatures. Hence, we could not today find any hydrogen or deuterium whose ratio could be measured, unless the H^1 and H^2 have come from direct injection by the solar wind and/or solar flares. Moreover, it demands an even larger amount of energy from the sun, $O_E=9\times 10^{48}$ erg. This is not inconsistent, however, with the observation[39] that some T-Tauri stars are spewing out mass at the rate of $3\times 10^{-8}M_\odot$ per yr, so that even if the mean energy/nucleon were as low as 1 MeV, the output would be 6×10^{50} erg in 10^7 yr. Moreover, such a regime would explain why chondrules were once molten.

There are several ways in which we can try to get

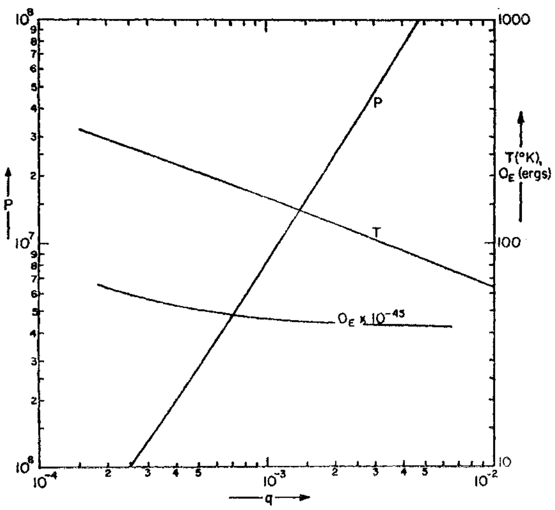

FIG. 14. The black-body temperatures of icy silicate No. 1 spheres, for $p=10^6$, as a function of the flare hardness parameter q. The intensity parameter p which will keep $T=273$°K for any value of q is also shown, as well as the maximum resulting solar energy out put in energetic protons alone (in ergs).

around this second difficulty. First, we may lengthen the time of irradiation. This would permit us to lower p, and hence the temperature. Secondly, we may suppose, as FGH did, that the proton spectrum from the primitive sun was much harder, which would raise the production. Third, we may include a considerable amount of carbon in our spheres—either elemental or as methane ice, etc., which will raise the mean production cross sections somewhat. Finally, the possibility exists that another constituent of the flares, particularly the α particles, may have been more effective in spallation than the protons. We will consider here only the second possibility.

In varying q, p is chosen at the largest value consistent with the constraint $T<273$°K. (The change in the freezing point for zero pressure is unimportant.) Suppose the thermal radiation alone yields 130°K. Then we must have

$$E_{\text{protons}}<\sigma[(273)^4-(130)^4]=3\times10^5\ \text{erg/cm}^2\ \text{sec}.$$

Thus, we want for icy silicate No. 1,

$$p<7.456\times10^{11}\alpha k q^{2/3},$$

and hence, inserting this and the expression for k into Eq. (37), the maximum energy required from the sun is

$$O_E<At\times7.456\times10^{11}(1+\alpha/\lambda q)$$
$$\cong4.2\times10^{46}(1+10^{-4}/q)\text{erg}.$$

In Fig. 14, p and O_E are plotted for icy silicate No. 1. Thus the energy required is two orders of magnitude smaller than before, though still quite high. The energy required is only $\sim10^{-2}$ times the gravitational energy released in collapsing to its present size. The spallation cross sections also change with q, and are shown in Fig. 15 for Be^7 off various targets. For Si^{28}, production rises faster with energy than for O^{16} and hence is included. A similar, perhaps slightly larger amount must hold for Mg^{24} and also for Al^{27}. Thus, as the mean proton energy increases, the silicon, aluminum, and magnesium begin to make a small but increasingly nonnegligible contribution to the effective spallation cross sections. Thus for icy silicate No. 1,

$$\sigma_s(Be^7)=(1/18.52)[7.88\sigma_s(Be^7,O^{16})+2.32\sigma_s(Be^7,Si^{28})].$$

The cross sections for the other isotopes are taken in the same proportion to $\sigma_s(Be^7)$ as in Table I.

Finally, the neutron production cross section also varies with q, according to Eq. (29). This is also plotted in Fig. 15, for the several compositions we consider.

The resulting isotopic ratios for icy silicate No. 1 as a function of q are shown in Fig. 16. The calculated ratios are consistent with the measured meteoritic ones at $q=8.8\times10^{-4}$, corresponding to $p=6.8\times10^6$.

Since we keep $t\cong10^7$ yrs constant, and vary p as so to keep $T<273$°K, this is effectively a one-parameter theory for the isotopic ratios in terms of q. To get the abundances we need a second parameter, the mean radius R.

The absolute concentrations are given in Fig. 17. The early part of the graphs have moved over, relative to

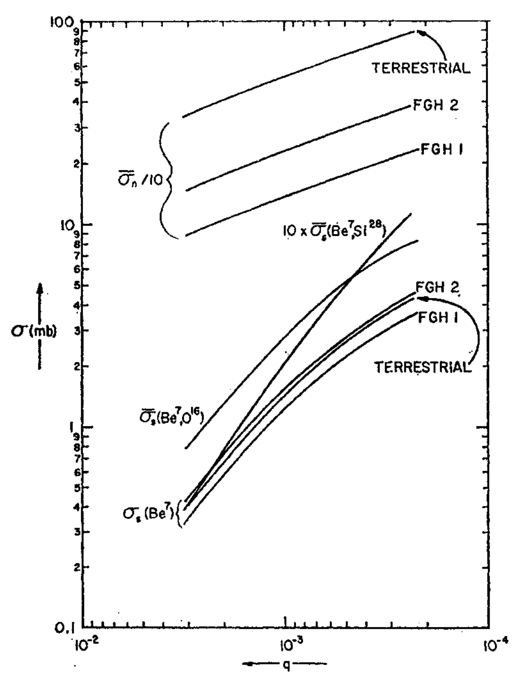

FIG. 15. The effective Be^7 spallation cross sections off O^{16} and Si^{28} are shown as functions of q, and the resulting $\bar{\sigma}_s(Be^7)$ for the three mixtures we consider: icy silicates Nos. 1 and 2, labeled FGH 1 and FGH 2, and the dry terrestrial mixture. The same is done with the neutron production cross sections.

Fig. 9, because the "skin" thickness is now greater, due to the harder flux.

From the asymptotic values of $R\bar{N}_a$ read off from this graph and the experimental meteoritic abundances $\bar{N}_a$, we find a different radius for each isotope:

$$\mathrm{Li}^6 \rightarrow R = 8.3\times10^4/3.1 = 268\ \mathrm{m}\,,$$
$$\mathrm{Li}^7 \rightarrow R = 1.0\times10^6/33 = 303\ \mathrm{m}\,,$$
$$\mathrm{Be}^9 \rightarrow R = 9.9\times10^4/3.4 = 290\ \mathrm{m}\,,$$
$$\mathrm{B}^{10} \rightarrow R = 4.3\times10^4/1.6 = 267\ \mathrm{m}\,,$$
$$\mathrm{B}^{11} \rightarrow R = 1.8\times10^5/6.1 = 296\ \mathrm{m}\,.$$

These values are sufficiently close that a single-size model will work. Thus, icy silicate No. 1 spheres, with the mean radius

$$\bar{R}_m = 285\pm18\ \mathrm{m}\,,$$

irradiated for 10^7 years with a mean proton flux $\varphi_p = 2p/3q = 5.1\times10^9$ protons/cm^2 sec, will yield the observed light-element abundances.

With about 10 ± 1 times the absolute abundance for terrestrial material, the spheres from which the earth agglomerated would have had the mean radius

$$\bar{R}_e = 29\pm3\ \mathrm{m}.$$

We are making the explicit assumption here that the material in these bodies mixed very thoroughly subsequent to their hypothesized agglomeration. Otherwise, it would be difficult to make any arguments at all about mean compositions. For the earth, moreover, this is especially reasonable, since all rocks manifest metamorphism, and the oldest rocks whose ages are measurable are only 3×10^9 yr old, whereas the age of the earth is now estimated at 4.5×10^9 yr.

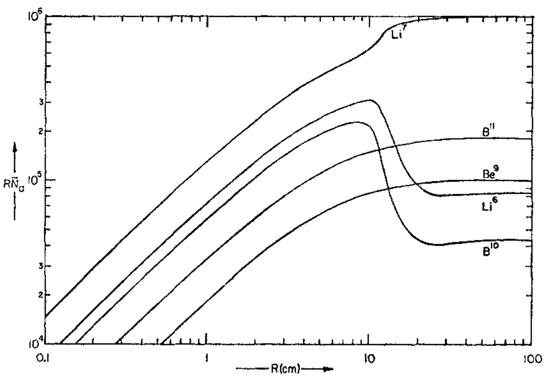

FIG. 17. Calculated (and normalized) abundances of spallated nuclei as a function of the radius, for icy silicate No. 1 spheres. These have been subjected to the harder flux ($q=7.8\times10^{-4}$) and an intensity which keeps their black-body temperature at 273°K ($p=5.6\times10^6$).

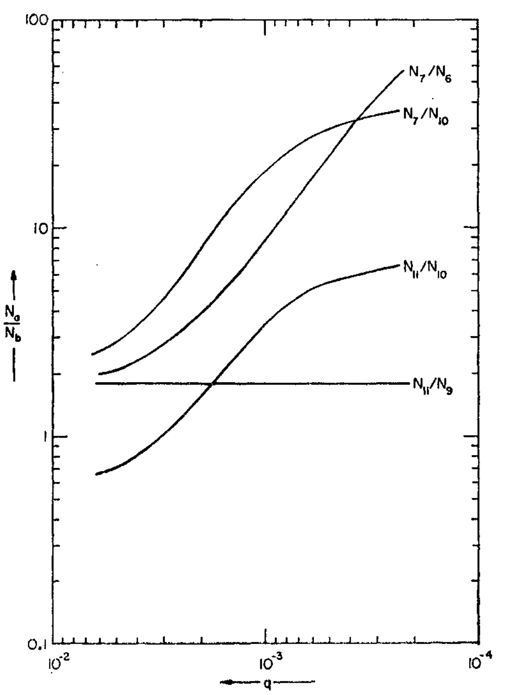

FIG. 16. The light-isotope ratios, as a function of the proton hardness parameter q. The intensity, as given by p, is varied so as to keep the temperature of the body at 273°K. The calculation is made for icy silicate No. 1 spheres of radius $R=80$ cm.

It is easily seen that $\bar{R}$ really is only a *mean* radius, for most generally, we must have

$$\sum_i n_i V_i \bar{N}_a(R_i) = \bar{N}_a(\mathrm{obs}) \sum_i n_i V_i\,,$$

where n_i is the number of spheres of radius R_i. If the distribution is approximated by a continuous one,

$$\bar{N}_a(\mathrm{obs}) = \int_0^\infty w(R) R^3 \bar{N}_a(R) dR \Big/ \int_0^\infty w(R) R^3 dR\,,$$

where $w(R)$ is the distribution function. Consider the distribution

$$w(R) = A e^{-R/R_0}.$$

We then find that we only need the asymptotic values of $\bar{N}_a$ for *large* spheres to determine R_0:

$$R_0 = \lim_{R\to\infty} \frac{R\bar{N}_a(R)_{\mathrm{calc}}}{3N_{a\ \mathrm{exp}}}\,,$$

so that

$$R_0 = \tfrac{1}{3} R_{m,e}\,,$$

where $R_{m,e}$ is the (single-size) radius of protoasteroidal or prototerrestrial spheres, respectively.

Thus we have arrived at a consistent solution for Li, Be, and B using icy silicate No. 1 spheres, with a relatively reasonable proton flux. Before going on to consider other possible planetesimal compositions, we must find the D/H ratio given by this flux. The contribution to the D/H ratio from spallation is shown in Fig. 18 as a dashed line. That from the $n(p,d)\gamma$ reaction, as a dotted line, and their sum, the total D/H ratio as a

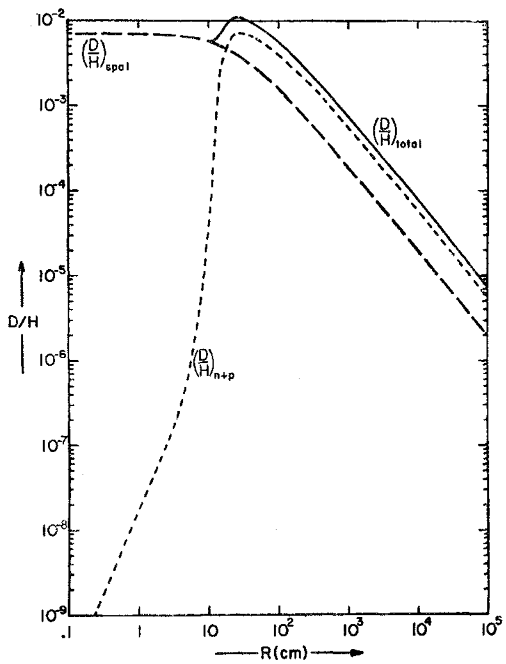

Fig. 18. The deuterium/hydrogen ratio as a function of sphere radius. The composition is icy silicate No. 1, and the bodies are irradiated in the flux $\varphi_p = 5.1\times10^9$ per cm² sec of 170-MeV protons. The dashed curve is the contribution from spallation, the dotted curve that from the $n(p,d)\gamma$ reaction, and the solid one is the sum of the two.

function of radius, is shown as a solid line. We see that for large R, D/H falls asymptotically as $1/R$, because the production is only a surface phenomenon. From this figure, we find that these sizes give rise to the deuterium/hydrogen ratios

$$\bar{R}_m = 285\ \text{m} \rightarrow (\text{D/H})_m = 2.7\times10^{-5},$$

and

$$\bar{R}_e = 29\pm3 \rightarrow (\text{D/H})_e = (2.7\pm0.3)\times10^{-4},$$

so that the order of magnitude difference in crustal and meteoritic light-element abundances leads to the same difference in D/H ratio. Experimentally, however, the *same* value is found for both terrestrial and meteoritic waters, $\text{D/H} = 1.56\times10^{-4}$. Thus the single-size model cannot account for this, unless (a) the water found in chondrites is largely contamination by terrestrial water; or (b) the D and H in the earth and in meteorites come from direct injection by the solar wind or flares; or (c) D and H are both primeval; or (d) the light elements have been concentrated tenfold in the crust.

If we believe the D/H measurements to be valid, on the other hand, we can try to explain the disparate results by adding a parameter, i.e., going to the two-size (cometesimal bombardment) model. Thus, suppose we had two sizes: one large sphere of radius $R_L > \bar{R}$, and a number n of smaller spheres of radius $R_s < \bar{R}$. We must have

$$V_s\bar{N}_a(R_s) + V_L\bar{N}_a(R_L) = (V_s + V_L)\bar{N}_a(\text{observed}), \quad (38)$$

where

$$V_L = \text{volume of large sphere}$$

and

$$V_s = \text{total volume of small spheres} = n\ (4\pi/3)R_s^3.$$

The D/H ratios are now made equal *ab initio* by taking R_s the same for asteroids and for the earth, while if the 10:1 ratio in absolute abundances is real, it can be attributed to a greater relative number of cometesimals captured by the earth. Call $R_L/R_s = x$. Then for the earth, Eq. (38) becomes (with R_s in meters)

$$[(n+x^3)/(n+x^2)]R_s = 29\pm3.$$

The large objects are assumed to be unirradiated and to contain neither H^1 nor H^2. Thus, the D/H ratio gives us R_s. From Fig. 18, we see that the observed D/H ratio occurs at $R_s = 50$ m. Now

$$(n+x^3)/(n+x^2) = b \equiv \bar{R}/R_s$$

implies

$$n = x^2(b-x)/(1-b).$$

For meteorites (i.e., asteroids) $b = 285/50 \cong 5.7$, and

$$n = x^2(x-5.7)/4.7,$$

so that solutions are possible for $x > 5.7$. For the earth, however, $b \cong 29/50 < 1$, and since we must have $x > 1$, *no solution is possible for the earth* for the two-size model, either, *unless we assume that the light elements have been concentrated in the crust.* The continuous (exponential) distribution can be no better: both the light-element abundances and the D/H ratio vary as $1/R$, so that if they are not consistent for one radius, they will be consistent for none.

Finally, we ask whether any of the naturally occurring isotopes are significantly depleted by the neutron flux. Gd^{157} has the largest absorption cross section among the stable isotopes, 2.4×10^5 b, and hence will

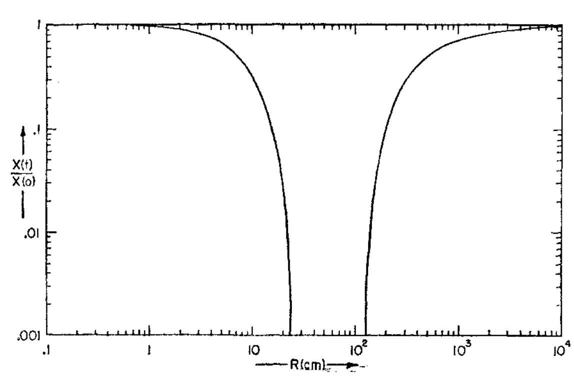

Fig. 19. Fraction of Gd^{157} in icy silicate No. 2 spheres, surviving 10^7 years' irradiation by solar protons, as a function of the sphere radius. $p = 2.48\times10^6$, $q = 0.00046$, $T = 273$°K.

give us the most sensitive test. It is observed in nature in approximately the relative abundance that one would expect—i.e., being even-odd, it is less (but not much less) abundant than its even-even neighbors, $Gd^{156,158}$. Thus it probably has been depleted negligibly (if at all). The fraction of Gd^{157} surviving after 10^7 yr of bombardment is plotted for FGH-2 spheres in Fig. 19, as a function of radius. The dependence for FGH 1 is very similar. From Eq. (35) we see that for the 290-m sphere, 99% of the Gd^{157} will survive. For the 29-m (proto-terrestrial) spheres, 90% will survive. For the two-size model, the large sphere has essentially no depletion at all, so that the net survival is >90%. For the one-size model, on the other hand, this predicts that the meteorites should have a 10% greater Gd^{157} relative abundance than the earth. This is inconsistent with the observation of Murthy and Schmitt[40] that there is no difference in terrestrial and in meteoritic isotope distribution of the Gd isotopes, within 1%. Hence, unless the model is quite wrong, this seems to argue that the high *Li, Be, and B abundances* measured in the crust are unrepresentative of the rest of the earth. Indeed, again, that they *have been concentrated some tenfold in the crust.*

The variation of $\bar{N}_a$ with time is shown in Fig. 20 for a body of icy silicate No. 1 composition and 80-cm radius. The irradiation is held at a value such that the H_2O will just remain as ice. We see that Be^9 and B^{11} increase linearly with time, as of course they should. Li^6

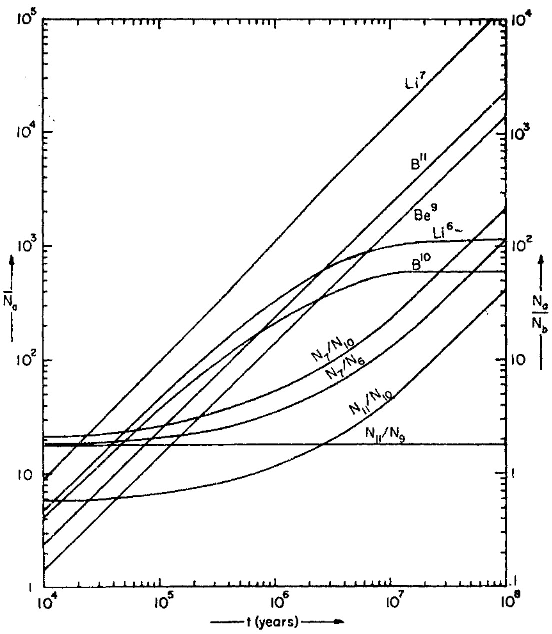

FIG. 20. Variation of isotopic abundance and their ratios for the spallated elements, as a function of time. The calculation is made for 80-cm spheres of icy silicate No. 1, with $q=0.00078$, $p=5.6\times10^6$, and $T=273°$K.

[40] V. R. Murthy and R. A. Schmitt, J. Geophys. Res. 68, 911 (1963).

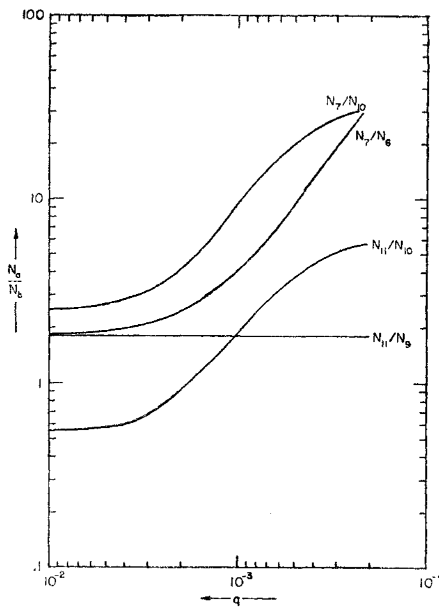

FIG. 21. The light-isotope ratios as a function of q for 80-cm spheres of icy silicate No. 2. p is varied so as to keep $T=273°$K.

and B^{10} are indistinguishable from their asymptotic values beyond $t=2\times10^7$ years. The deviation of Li^7 from a linear increase with time is almost imperceptible. Indeed, beyond 2×10^7 yr, the increase must again be linear, as *all* of the new B^{10} being produced is now going into Li^7. Since all the ratios (except B^{11}/Be^9) vary together, the results are roughly invariant to changes in t, so long as $\varphi_p t$ is kept constant.

We can now go on to icy silicate No. 2. We calculate the isotopic ratios as a function of q, as before. These are shown in Fig. 21. It is very similar to Fig. 16, but translated to the right, so that a somewhat harder flux is necessary.

From Fig. 21, we find a fairly consistent solution at $q=4.6\times10^{-4}$. Thus, we are indeed forced toward Fowler's initial assumption, that the proton flux was harder than it is today, with mean energy $\bar{E}\cong250$ MeV. For this value of q, the asymptotic values of $R\bar{N}_a$ again give consistent radii, and

$$\bar{R}_m=225\pm6\text{ m}.$$

The graph of D/H versus R is very similar to the one for icy silicate No. 1, the principal difference being that the contribution to D/H from spallation is about 2.5 times as large as before, so that the observed D/H ratio now occurs at $R=85$ m. Thus, we still do not quite have consistency. The completely dry terrestrial material yields almost exactly the same curves, as was the case before, but translated a bit further still, so that the solution lies at $q=3.45\times10^{-4}$. This corresponds to $p=1.61\times10^6$, or a flux $\varphi_p=3.1\times10^9$ protons/cm² sec of protons of mean energy $\bar{E}=300$ MeV. The mean radius is $\bar{R}_m=137\pm6$ m. The spallation contribution to

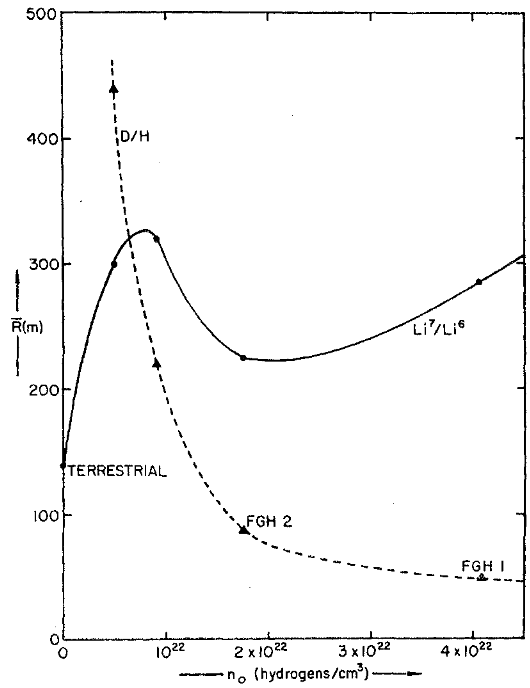

FIG. 22. The mean radius $\bar{R}$ giving the observed isotopic ratios, as a function of the initial hydrogen concentration per cm^3. The dashed line gives the radii yielding $H/D=6400$, while the solid line gives R such that $Li^7/Li^6=10.5$. There is a consistent solution at $n_0=6\times10^{21}$, for $R=320$ m. This is, of course, only approximate, as the calculations with $0\leq n_0<10^{22}$ have been made with Fe in the composition, while those for FGH 1 and FGH 2 have none.

D/H is the principal one, and $D/H=3.66\times10^{-2}$ for all R. Thus we *must* add some hydrogen to lower this value, i.e., dilute the spallated deuterium. By trying a number of compositions, we find the radii which give the observed D/H and light-element isotopic ratios, as a function of n_0. This is shown in Fig. 22. It is clear from this figure that a consistent solution occurs at $n_0=6\times10^{21}$, corresponding to the addition of about $N(H_2O)=0.24$ to the terrestrial composition. This is just 10% H_2O by volume. If we wish, we may include some of the H_2O in the form of water of hydration. The bodies have a mean radius of 320 m, and $q=1.35\times10^{-4}$, corresponding to a mean energy of 570 MeV, just about the energy posited by FGH. The proton flux is 2×10^9 sec^{-1} cm^{-2}.

The principal difficulty now lies in getting rid of the considerable amounts of H_2O in the body. Of course, if the body goes through a heating cycle, it will lose the surface hydrogen first, so that a smaller dilution would be necessary. But it would then be difficult to understand how both prototerrestrial and protoasteroidal planetesimals went through such similar heating cycles.

If we did not require the correct D/H ratio, we conclude that we can obtain the Li, Be, and B abundances with objects of almost any silicaceous composition, including quite dry ones.

IX. ASTROPHYSICAL SPECULATIONS

Let us compare the one- and two-size models from a different point of view. If we assume Hoyle's picture of protons leaving the sun along the magnetic lines of force of a distorted and wound-up solar dipole field, we find that at near distances, the flux will vary as r^{-3}, while in the gaseous disk, it will vary roughly as r^{-1}. In the intermediate region, we may take $p\sim r^{-2}$.

We assume that the asteroids are the source of meteorites. According to Bode's law, the mean distance of the asteroids from the sun is 2.8 au (astronomical units). Hence, if the intensity of the proton flux falls off as the inverse square of the distance from the sun, $p(\text{earth})\cong8p(\text{asteroids})$. We then find that (as long as the bodies were of dimensions >50 cm) we would obtain the terrestrial ratio

$$N_7/N_6\cong72.$$

Even if we only have the flux falling off as r^{-1} in the disk, and suppose that the asteroids giving rise to meteors are only those within Mars' orbit, we still have

$$p(\text{earth})\cong1.5\,p(\text{asteroid})$$

which implies that we should obtain the terrestrial ratio

$$N_7/N_6\cong1.4N_7/N_6(\text{asteroid})=1.47.$$

In fact, however, from Honda and Shima's data,[34] we find that the Li^7/Li^6 ratio goes from 10.5 for the asteroid fragments to 12.0 for terrestrial rocks—only a 14% enhancement. This corresponds to an increase in p of 19%, which corresponds to raising the B^{11}/B^{10} ratio from 4.05 to 4.33, i.e., a 6.9% increase. This is in excellent accord with the 6.5% enhancement measured by Shima,[35] i.e., from 3.85 to 4.1.

If we attribute the difference in p to differences in the bodies' mean distances from the sun during irradiation, we see that they must have then been much closer together. This could only have been so if they had all been much closer to the sun. This, however, would require the subsequent movement of these large bodies away from the sun through very large distances, for which there seems to be no adequate mechanism.

On the other hand, if cometesimals were irradiated, then we should expect a much smaller difference in the ratios, assuming that the mean perihelia of the subsequently captured cometesimals were roughly the same for the earth as for the asteroids. The absolute abundances would depend on the relative number captured and on R_L (asteroid). Because of the small masses of the asteroids, we would expect a small number to be captured and hence a small absolute abundance. However, we can make no quantitative estimates. Let us assume, then, that the planetesimals were already distributed in belts near their present orbits, and irradiated there.

If the magnetic fields were largely disordered in the disk, then the protons from flares would essentially *diffuse* away from the sun, and if we assume cylindrical symmetry, we find the steady-state flux distribution to be

$$\varphi_p(r) \cong \frac{S_1}{2R_\odot} \frac{I_0(\kappa R_\odot)}{2\pi D} K_0(\kappa r),$$

where D is the diffusion coefficient for the protons, $\kappa^2 = \sum_a/D$, and S_1 is the output from the sun in protons/sec. We can find a κ such that the flux at $r=1$ au astronomical unit is 1.19 times that at 2.8 au; indeed, this gives $\kappa^{-1}=560$ a.u. However, this again leads to very large energy requirements from the sun in protons alone: a minimum of 4×10^{47} erg.

We can, however, explain the near identity of the terrestrial and meteoritic isotope ratios with the following model: The intense solar flares were accompanied by plasma clouds which were emitted preferentially along the ecliptic, and penetrated any solar magnetic fields. Each flare cloud contained its own magnetic fields which stored the high-energy protons within the cloud for appreciable times. The cloud expanded by diffusion as it traveled away from the sun. Contrary to the situation in the presently outgassed solar system, however, it could only expand until its density approached that of the ambient gas. Beyond that point it continued to travel out, with the trapped proton flux now undiminished, except for diffusion out of the cloud and absorption by intervening matter. Assume that loss by diffusion was negligible, and that the cloud had stopped expanding by the time it arrived near the earth's orbit. In order for the intensity at the earth to be 19% greater than at the asteroid belt, then, the prototerrestrial and protomartian bodies must have absorbed 16% of the protons (on the average) from the flare couds. The total number of protons required per sphere is $\varphi_p \pi R^2 t$. We need $(R_e/R)^3 \rho_e/\rho$ spheres to accrete into the earth.

If we assume that the flare clouds stopped expanding by the time they reached Mercury's orbit, we can easily find the mean flux each belt of planetesimals must have been subjected to. Assume that there has been no mass loss from any of the belts, in accreting to the present planets.

We then find that the flux of protons in the cloud at the nth belt is given by

$$\varphi_n(t) = \varphi_n(0)\exp[-3M_n\bar{v}t/4R_n\rho_n\Omega],$$

where Ω is the cloud volume, $\bar{v}$ the rms proton velocity, M_n the planetary mass, and R_n, ρ_n the planetesimal radius and density, respectively.

Thus the mean flux is given by

$$\bar{\varphi}_n = [\varphi_n(0)/a_n](1-e^{-a_n}),$$

where $a_n \equiv 3M_n\bar{v}t_n/4R_n\rho_n\Omega$, t_n being the mean transit time for the cloud. Moreover, it is easy to show that

$$\varphi_n(0) = \varphi_0 \exp\left(-\sum_{i=1}^{n-1} a_i\right),$$

Table IX. The proton fluxes the planetesimals were subjected to, and the resulting lithium isotope ratios. The second column gives the total area irradiated by the protons. The planetesimals for the asteroids and minor planets are taken to be 320-m spheres of terrestrial composition with 10% water. Those for the major planets are taken to be largely ice.

Planet	$3M/4R\rho$ (cm^2)	$\sum_{i=1}^{n} a_i$	$\bar{\varphi}_p \times 10^{-9}$	Li^7/Li^6
Mercury	1.9×10^{21}	0.015	3.5	16.4
Venus	2.8×10^{22}	0.24	3.1	14.9
Earth	3.5×10^{22}	0.52	2.4	12.1
Mars	3.8×10^{21}	0.55	2.1	10.7
Asteroids	1.8×10^{19}	0.55	2.0	10.5
Jupiter	2.8×10^{25}	220.	0.0092	1.92
Saturn	8.3×10^{24}	290.	0	1.85
Uranus	1.3×10^{24}	300.	0	1.85
Neptune	1.5×10^{24}	310.	0	1.85

φ_0 being the original flux impinging on the protomercurial belt. If all the transit times are assumed equal to t_0, and if we assume that the inner planets accreted from planetesimals of 320-m mean radius and hydrated terrestrial composition (i.e., 10%H_2O), then $\bar{\varphi}_3/\bar{\varphi}_5 \cong 1.19$ leads to

$$\bar{v}t_0/\Omega = 8.4\times10^{-24}\ \text{cm}^{-2}.$$

Moreover, with $\bar{\varphi}$ (asteroid) $=\bar{\varphi}_5 = 2.0\times10^9$, we find $\varphi_0 = 3.5\times10^9$ nucleons/cm^2 sec.

If we further assume that the major planets' planetesimals were largely icy, and of mean radius 300 m, mean density ~1, then we can find the mean fluxes experienced by each planet, and the resultant Li^7/Li^6 ratio to be expected. These are given in Table IX. We note that Jupiter absorbed essentially all the remaining protons (58%), and we therefore expect a negligible concentration of the light elements on Saturn, Uranus, and Neptune. Whatever *is* there, however, is just in the ratio of the spallation cross sections. In 10^7 yr the prototerrestrial bodies absorbed 2.1×10^{46} protons, and this is 15% of all protons. Hence the total solar output of protons in flare clouds, assuming that all the flares were intercepted, must have been 1.4×10^{47} protons. At 570-MeV mean energy, this corresponds to 1.3×10^{44} ergs. This, finally, is a reasonable energy. If the sun were emitting energy faster than this, than all or nearly all of the ices must have been removed from the original spheres. Unless the temperature was driven above 450°K, however, most of the water of hydration would have been undisturbed.

Finally, recent work on the nature of the moon's surface[41] suggests that these spheres might have accreted as rather fluffy structures. This idea has the merit that such bodies would have lower velocities relative to each other in the formative nebula, from Stoke's law. Moreover, when there *is* a collision, they would be able to transform some of the kinetic energy

[41] B. Hapke and H. Van Horn, J. Geophys. Res. **68**, 4545 (1963); B. W. Hapke, *ibid.* **68**, 4571 (1963).

of impact more readily into heat which can be radiated away, than would similarly massive, but more compact bodies.

X. CONCLUSIONS AND RESUMÉ

1. The spallation cross sections, except possibly for lithium, are not known well enough to prove conclusively that this theory of the origin of the light elements is consistent with the data.

2. Contrary to expectations, the calculated ratios are very insensitive to the composition of the irradiated planetesimals, demanding only slightly different fluxes to give the same results. On the other hand, this makes it easy to contemplate a process in which the bodies within Jupiter's orbit initially had much ice, but lost it by evaporation. The H_2O in the icy silicate mixture can then be taken to be water of hydration in a terrestrial mix, rather than ice, as long as the temperature does not rise so high as to drive it off. With no bound hydrogen at all, the resulting D/H ratio would be $\sim 10^2$ times what is observed. If we demand the observed D/H ratio as well as the others, then, this fixes the H_2O at about 10% by abundance.

3. We cannot determine a size spectrum for the protoplanetary bodies at all. Only one parameter can be extracted: a mean radius for each planet.

4. The mean size of the bodies is also rather insensitive to the composition, varying from 140–350-m mean radius for the protoasteroidal bodies.

5. In order to keep the planetesimals relatively cool and keep the energy demand from the sun reasonable, we are forced to assume a considerably harder flare proton energy spectrum than is observed today.

6. The meteoritic and terrestrial isotope ratios are nearly identical, which implies that their planetesimals were subjected to very similar fluxes.

7. Two simple models are considered:

a. A two-size model, where the small bodies are cometesimals passing close to the sun, of radius R_s such that we get the observed D/H ratio. The large bodies were supposed to be at their present orbits, and were showered by the irradiated cometesimals. This model can be made to work for the asteroids, but not for the earth, unless we assume that the light elements have been concentrated in the crust by an order of magnitude.

b. A single-size model. The only reasonable way to explain point No. 6, assuming that the planetesimals were in belts at roughly the present orbits during the period of irradiation, is to assume that the plasma clouds from solar flares ceased to expand beyond earth's orbit.

8. For the single-size model, the apparent difference in abundance of the light elements between earth and asteroids may be attributed to different sphere sizes. This immediately leads to different D/H ratios, however, as well as a 10–20% Gd^{157} abundance difference.

9. If the crustal abundances of the light elements are representative for the earth as a whole, therefore, a more likely source for the hydrogen and deuterium is direct injection by the solar wind and/or solar flares. It is also possible that both D and H are primeval, or that meteoritic water (even of hydration) has been exchanged with ambient terrestrial water.

10. On the other hand, the Gd^{157} and D/H inconsistencies disappear if we assume that there has been a tenfold concentration of Li, Be, B in the earth's crust.

11. If the plasma clouds from solar flares are assumed not to have continued expanding beyond Mercury's orbit, we can make an estimate of the varying proton fluxes and resulting Li^7/Li^6 ratios for all the planets.

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